Spectrum Leasing to Cooperating Secondary Ad Hoc Networks

O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz

***Abstract*—The concept of cognitive radio (or secondary spectrum access) is currently under investigation as a promising paradigm to achieve efficient use of the frequency resource by allowing the coexistence of licensed (primary) and unlicensed (secondary) users in the same bandwidth. According to the property-rights model of cognitive radio, the primary terminals own a given bandwidth and may decide to lease it for a fraction of time to secondary nodes in exchange for appropriate remuneration. In this paper, we propose and analyze an implementation of this framework, whereby a primary link has the possibility to lease the owned spectrum to an ad hoc network of secondary nodes in exchange for cooperation in the form of distributed space-time coding. On one hand, the primary link attempts to maximize its quality of service in terms of either rate or probability of outage, accounting for the possible contribution from cooperation. On the other hand, nodes in the secondary ad hoc network compete among themselves for transmission within the leased time-slot following a distributed power control mechanism. The investigated model is conveniently cast in the framework of Stackelberg games. We consider both a baseline scenario with full channel state information and information-theoretic transmission strategies, and a more practical model with long-term channel state information and randomized distributed space-time coding. Analysis and numerical results show that spectrum leasing based on trading secondary spectrum access for cooperation is a promising framework for cognitive radio.**

***Index Terms*—Cognitive radio, property-rights, Stackelberg games, cooperative transmission, spectrum leasing.**

I. INTRODUCTION

The lively debate around the concept of cognitive radio has by now broadened its scope to include substantially different technologies and solutions. The identifying feature, which seems to be common to different schools of thought on the subject, is the coexistence on the same spectral resource of both licensed (or primary) and unlicensed (or secondary) terminals and services [1]. The very *raison d'être* of cognitive radio is in fact the evidence that: (*i*) current spectrum allocation granting exclusive use to licensed services is highly inefficient, (*ii*) new wireless communication technologies allow effective spectrum sharing. This principle has already inspired technological solutions [2] and standardization efforts [3] [4].

Manuscript submitted: Feb. 26, Revised: Sept. 27.

O. Simeone, I. Stanojev, and Y. Bar-Ness are with the Center for Wireless Communications and Signal Processing Research, New Jersey Institute of Technology, Newark, New Jersey 07102-1982, USA (e-mail: osvaldo.simeone@njit.edu, iss2@njit.edu barness@yegal.njit.edu).

S. Savazzi and U. Spagnolini are with Dipartimento di Elettronica e Informazione, Politecnico di Milano, Milan, I-20133 Italy (e-mail: savazzi, spagnoli@elet.polimi.it).

R. Pickholtz is with the Department of Electrical Engineering and Computer Science, The George Washington University, Washington D.C. 20052 (e-mail: pickholt@gwu.edu).

Digital Object Identifier 10.1109/JSAC.2008.0801xx.

Among the different debated positions, two main approaches to cognitive radio have emerged [1] [5] [6]:

- *Commons* model: according to this framework, primary terminals are oblivious to the presence of secondary users, thus behaving as if no secondary activity was present. Secondary users, instead, sense the radio environment in search of spectrum holes (portions of the bandwidth where primary users are not active) and then exploit the detected transmission opportunities.
- *Property-rights* model (or *spectrum leasing*): here, primary users own the spectral resource and possibly decide to lease part of it to secondary users in exchange for appropriate remuneration.

The commons model is currently being studied using different tools such as information theory [7] [8], partially observable Markov chains [9] and queuing theory [10]. The property-rights model has instead been seldom analyzed in the communication engineering literature on the grounds that its implementation is mostly a regulatory issue that hinges on the definition of a pricing model for spectrum leasing [5].

A. Cooperation-based spectrum leasing

In this paper, we focus on the property-rights model and propose a framework in which secondary terminals are granted the use of a given spectral resource by the incumbent primary in exchange for cooperation. The rationale is that the primary nodes will be willing to lease the owned bandwidth for a fraction of time if, in exchange for this concession, they will benefit from enhanced quality of service (e.g., in terms of rate or probability of outage) thanks to cooperation with the secondary nodes. In turn, the secondary nodes have the choice about whether to cooperate or not with the primary on the basis of the amount of cooperation required by the primary and the corresponding fraction of time leased for secondary transmissions.

We consider a single primary link sharing the spectrum with an ad hoc network of secondary nodes as depicted in Fig. **??**. In the secondary network, each transmitter has information to deliver to a given secondary receiver (interference channel). The primary link may lease the owned bandwidth for a fraction of time to a subset of secondary transmitters in exchange for cooperation (relaying) in the form of transmission via distributed space-time coding (DSTC) [11].

In the fraction of time available for the secondary activity, the subset of selected secondary transmitters compete for transmission to their respective receivers by performing decentralized power control. Accordingly, each secondary node seeks to maximize a utility function that accounts for the cost/

benefits trade-off between the expense in terms of transmitted power and the achievable quality of service. For instance, an excessively small fraction of time leased for secondary transmission may not compensate for the overall cost of transmission (including relaying) and the secondary may then decide not to transmit, and thus not cooperate with the primary. The outcome of this competitive and decentralized behavior can be described by the basic solution concept in game theory, namely the *Nash Equilibrium* (NE) [12]. Game-theoretic analysis of the interference channel with distributed power control has been reported in [13] [14].

Overall, the considered framework is characterized by a hierarchical structure, where one agent (the primary link) optimizes its strategy (leased time and amount of cooperation) based on the knowledge of the effects of its decision on the behavior of a second agent (the secondary network). A convenient analytical model to study this scenario is provided by *Stackelberg games* [12]. Related work on application of Stackelberg games to wireless communications can be found in [15], where the main focus is promoting cooperation in ad hoc networks, and in [16], where the authors are primarily interested in the optimal design of an access point in a decentralized network.

II. System Model

In the following, we detail the model of spectrum leasing based on cooperation and the main system parameters.

A. Medium access control (MAC) layer

We consider the system sketched in Fig. 1, where a primary (licensed) transmitter PT communicates with the intended receiver PR in slots of N_{S} symbols. In the same bandwidth, a secondary (unlicensed) ad hoc network $\mathcal{S}_{tot}$, composed of K transmitters $\{\mathrm{ST}_i\}_{i=1}^K$ and K receivers $\{\mathrm{SR}_i\}_{i=1}^K$, is seeking to exploit possible transmission opportunities. We assume one-to-one communication in $\mathcal{S}_{tot}$, i.e., the data from the secondary terminal ST_i is intended for the secondary receiver SR_i (interference channel).

The primary transmitter PT grants the use of the bandwidth to a subset $\mathcal{S} \subseteq \mathcal{S}_{tot}$ of $|\mathcal{S}| = k \leq |\mathcal{S}_{tot}| = K$ secondary nodes in exchange for cooperation so as to improve the quality of the communication link to its receiver PR. In particular, the primary decides whether to use the entire slot for direct transmission to PR or to employ cooperation. In the latter case, a fraction $1-\alpha$ of the slot ($0 \leq \alpha \leq 1$) is used for transmission from PT to the secondary nodes in $\mathcal{S}$ (see Fig. 1-(a)). Moreover, the remaining αN_{S} symbols are further decomposed into two subslots according to a parameter $0 \leq \beta \leq 1$. The first subslot is of duration $\alpha\beta N_{\mathrm{S}}$ and is dedicated to cooperation: the set $\mathcal{S}$ of active ST_i form a distributed k-antenna array that cooperatively relay the primary codeword through DSTC towards PR [11] [17] (Fig. 1-(b)). In the last subslot of duration $\alpha(1-\beta)N_{\mathrm{S}}$ symbols, the active secondary nodes are allowed to transmit their own data (Fig. 1-(c)). In this last subslot, the transmissions scheme amounts to an interference channel with distributed power control [13] [14]. Finally, notice that the durations of different subslots are constrained to be integer numbers, which is guaranteed if αN_{S} and $\alpha\beta N_{\mathrm{S}}$ are integer.

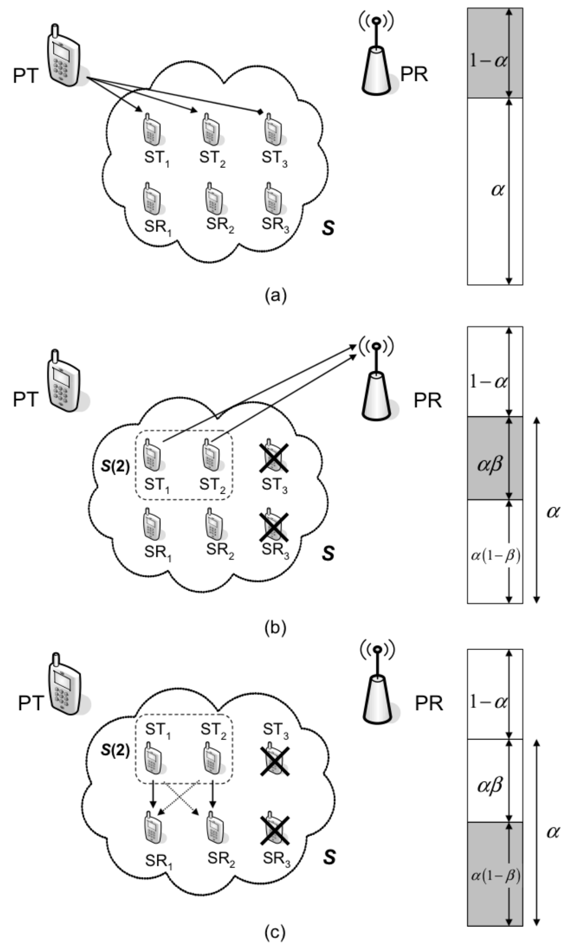


Fig. 1. Secondary spectrum access through cooperation-based spectrum leasing, for $K = 3$ secondary transmitters and receivers: (a) primary transmission; (b) space-time coded cooperation; (c) secondary transmission.

B. Physical layer

The channels between nodes are modeled as independent proper complex Gaussian random variables, invariant within each slot, but generally varying over the slots (i.e., Rayleigh block-fading channels). We use the following notation to denote the instantaneous fading channels in each block: h_{P} denotes the complex channel gain between primary transmitter PT and primary receiver PR; $h_{\mathrm{PS},i}$ the channel gain between PT and secondary transmitter ST_i; $h_{\mathrm{SP},i}$ between ST_i and PR; $h_{\mathrm{S},ij}$ between ST_j and SR_i for any $i, j = 1, ..., K$. Moreover, the average channel gains are denoted as: $\mathrm{E}[|h_{\mathrm{P}}|^2] = g_{\mathrm{P}}$, $\mathrm{E}[|h_{\mathrm{PS},i}|^2] = g_{\mathrm{PS},i}$, $\mathrm{E}[|h_{\mathrm{SP},i}|^2] = g_{\mathrm{SP},i}$ and $\mathrm{E}[|h_{\mathrm{S},ij}|^2] = g_{\mathrm{S},ij}$. We are interested in two scenarios characterized by different assumptions about the available Channel State Information (CSI) at different nodes and different constraints on the available transmission strategies:

1) *Instantaneous CSI* (Sec. III): in this baseline case, the primary link is assumed to be aware of the instantaneous realizations of all the fading power gains in the system (i.e., $|h_{\mathrm{P}}|^2$, $|h_{\mathrm{PS},i}|^2$, $|h_{\mathrm{SP},i}|^2$ and $|h_{\mathrm{S},ij}|^2$), while the secondary nodes are aware only of the fading power gains $|h_{\mathrm{S},ij}|^2$ within the secondary network. We notice that,

albeit ideal, the assumption of instantaneous CSI is very common in the literature on game-theoretic applications to wireless networks (see, e.g., [13] [14]) and provides a convenient framework for analysis. Moreover, cooperation via DSTC is studied by considering information theoretic achievable rates [11] [18].

2) *Long-term CSI* (Sec. IV): in this scenario, we remove the idealistic assumptions made in the baseline case discussed above. In particular, we assume that the primary link only knows the statistics of the fading channels in the whole system, while the secondary nodes are aware of the statistics of the fading channels within the secondary network. We remark that fading statistics are reasonably considered to be constant for a large number of transmission blocks, thus making the estimation of this CSI a feasible task. Furthermore, in order to account for more practical coding schemes, herein randomized cooperation among secondary terminals is deployed using DSTC as proposed in [17].

The transmission power of the primary is denoted as $P_{\rm P}$. On the other hand, the ith secondary node in the active subset $\mathcal{S}$ transmits with power $0 \leq \hat{P}_i \leq P_{\max}$, where the powers, collected in a $k \times 1$ vector $\hat{\mathbf{P}} = [\hat{P}_i]_{i\in\mathcal{S}}$, are obtained as the outcome (NE) of the power control game played between secondary nodes in the last subslot of $\alpha(1-\beta)N_{\rm S}$ symbols. Notice that each of the k activated secondary nodes in $\mathcal{S}$ is constrained to use the same power for both cooperation and own traffic (see last two subslots in Fig. 1-(b) and Fig. 1-(c)). This rule is enforced in order to ensure that the secondary nodes spend for cooperation the same power that they are willing to spend for their own transmission[1]. Finally, the single-sided spectral density of the independent white Gaussian noise at the (both primary and secondary) receivers is N_0.

C. A note on the protocol

Spectrum leasing requires the primary link to be aware of the number and identity of secondary nodes available in the band of interest. In practice, this entails the need for a control channel to be used, e.g., for communicating the availability of a new secondary user to the primary. This control channel can also be used for the exchange of CSI parameters among different nodes (e.g., between secondary network and primary transmitter) and for delivering the decision of the primary (slot allocation and cooperation parameters[2]) to the secondary network. The need for a control channel, and the consequent reduction in spectral efficiency, is the price to be paid for the implementation of spectrum leasing. This contrasts with the *commons model*, where the primary is oblivious to the presence of the secondary nodes and no control channel needs to be set up.

[1]A more general model would have the power used for cooperation to be an increasing function of the power used for own transmission. As explained in Sec. III-B, our treatment readily applies to the case where this function is linear.

[2]As it will be made clear in the following, in the case of instantaneous CSI the primary directly selects the set of cooperating secondary nodes $\mathcal{S}$, whereas with long-term CSI the primary is only able to select the STC to be employed.

III. COOPERATION-BASED SPECTRUM LEASING WITH INSTANTANEOUS CSI

In this section, we describe and analyze spectrum leasing based on space-time coded cooperation within a Stackelberg game framework under the assumption of instantaneous CSI. As anticipated above, here we consider information theoretic achievable rates as performance measures and thus the number of symbols $N_{\rm S}$ per blocks is assumed to be very large ($N_{\rm S} \rightarrow \infty$). We first discuss the Stackelberg model of interaction between the primary link (leader) and competitive secondary network (follower) in Sec. III-A and III-B. Insight on the system performance is provided via analysis in Sec. III-C and numerical results in Sec. III-D.

A. Primary link

Based on the available instantaneous CSI, the primary link selects the slot allocation parameters (α,β) and the subset of cooperating secondary nodes $\mathcal{S} \subseteq \mathcal{S}_{tot}$ towards the aim of optimizing its transmission rate $R_{\rm P}(\alpha,\beta,\mathcal{S})$ (measured in bits/symbol or equivalently bits/s/Hz). To start with, in the baseline case where the fraction of time leased to $\mathcal{S}$ is $\alpha=0$, we set the primary transmission rate to $R_{\rm P}(0,\beta,\mathcal{S}) = R_{\rm dir}$, where $R_{\rm dir}$ is the rate on the direct link between PT and PR:

$$R_{\rm dir} = \log_2\left(1+\frac{|h_{\rm P}|^2 P_{\rm P}}{N_0}\right). \quad (1)$$

Instead, if the fraction of time leased to the subset $\mathcal{S}$ of k active secondary users is $\alpha > 0$, we assume the use of a collaborative scheme based on decode-and-forward multihop and space-time coding, for which the achievable primary rate reads[3] [11] [18]

$$R_{\rm coop}(\alpha,\beta,\mathcal{S}) = \min\{(1-\alpha)\, R_{\rm PS}(\mathcal{S}), \alpha\beta R_{\rm SP}(\alpha,\beta,\mathcal{S})\}, \quad (2)$$

as detailed below. The cooperative rate (2) (valid for $\alpha > 0$) is the minimum between two terms. The first term is the rate achievable in the first subslot (Fig. 1-(a)) between the primary transmitter PT and all the secondary transmitters in the subset $\mathcal{S}$. Recalling that secondary nodes cannot cooperate among themselves for detection, this rate is easily shown to be dominated by the worst channel $|h_{{\rm PS},i}|^2$ in the subset $i \in \mathcal{S}$ as

$$R_{\rm PS}(\mathcal{S}) = \log_2\left(1+\frac{\min_{i\in\mathcal{S}} |h_{{\rm PS},i}|^2 P_{\rm P}}{N_0}\right). \quad (3)$$

Notice that the factor $(1-\alpha)$ in the first term of (2) is due to the fraction of time occupied by the first slot. The second term in (2) is the achievable rate $\alpha\beta R_{\rm SP}(\alpha,\beta,\mathcal{S})$ between the

[3]Notice that larger achievable rates could be achieved by more sophisticated coding/ decoding schemes, e.g., coding by rate splitting (partial decoding) and decoding at the destination based on the signal received in both slots (instead of multihop) [18]. Here we focus on simple decode-and-forward multihop only for simplicity of presentation in order not to introduce inessential analytical complications. Moreover, it is noted that using other decode-and-forward or compress-and-forward [18] cooperative schemes would lead to the same main qualitative conclusions with the apparent caveat that using a better (resp. worse) cooperative transmission strategy would entail more (resp. less) time leased to the secondary network. Finally, it should be remarked that amplify-and-forward schemes do not provide the necessary flexibility in slot allocation needed in our scenario since they require equal-time transmission slots.

k secondary nodes in $\mathcal{S}$ and the primary receiver PR assuming DSTC cooperation (Fig. 1-(b)) [11]

$$R_{\mathrm{SP}}(\alpha,\beta,\mathcal{S}) = \log_2\left(1+\sum_{i\in\mathcal{S}}\frac{|h_{\mathrm{SP},i}|^2\,\hat{P}_i(\alpha,\beta,\mathcal{S})}{N_0}\right). \quad (4)$$

We remark that the rate (4) is obtained following the ideal information-theoretic assumption of employing orthogonal Space-Time Codes (STCs) that are able to harness the maximum degree of diversity from cooperation (see Sec. IV for a more practical approach)[4]. Moreover, the cooperation rate (4) depends on the powers $\hat{P}_i(\alpha,\beta,\mathcal{S})$ autonomously selected by the secondary transmitters, as the outcome (NE) of the non-cooperative power control game played by the secondary terminals in the third subslot (Fig. 1-(c)).

To summarize, from (1) and (2), the primary transmission rate $R_{\mathrm{P}}(\alpha,\beta,\mathcal{S})$ is given by

$$R_{\mathrm{P}}(\alpha,\beta,\mathcal{S}) = \begin{cases} R_{\mathrm{dir}} & \text{for } \alpha = 0 \\ R_{\mathrm{coop}}(\alpha,\beta,\mathcal{S}) & \text{for } \alpha > 0 \end{cases}. \quad (5)$$

and the primary link aims at solving the following rate-optimization problem:

$$\begin{array}{l} \max\limits_{\alpha,\beta,\mathcal{S}} R_{\mathrm{P}}(\alpha,\beta,\mathcal{S}) \\ \text{s.t. } \mathcal{S}\subseteq\mathcal{S}_{tot},\ 0\le\alpha,\beta\le 1 \end{array}. \quad (6)$$

Notice that the integer constraints on the number of symbols in each subslot can be neglected at this point given the assumption $N_{\mathrm{S}}\to\infty$ (see Sec. IV for analysis of a scenario with finite N_{S}). The optimization problem (6) can be interpreted as a Stackelberg game [12]: The primary link is the Stackelberg *leader*, that optimizes its strategy $(\alpha,\beta,\mathcal{S})$ in order to maximize its revenue according to (6), being aware that its decision will affect the strategy selected by the Stackelberg *follower* (the secondary ad hoc network), namely the set of transmitting powers $\hat{\mathbf{P}}(\alpha,\beta,\mathcal{S})$.

B. Ad hoc secondary network

Any active secondary terminal ST_i, with $i=1,...,k$, in the subset $\mathcal{S}$ attempts to maximize the rate towards its own receiver SR_i discounted by the overall cost of transmission power (as explained below), being aware of the parameters α,β and $\mathcal{S}$ selected by the primary and acting in a rational and selfish way (see, e.g., [12] for rigorous definitions of these terms). In particular, each secondary transmitter ST_i with $i\in\mathcal{S}$ chooses its transmitting power P_i according to the NE $\hat{P}_i(\alpha,\beta,\mathcal{S})$ (we will show that it exists and is unique) of the non-cooperative power control game $\langle\mathcal{S},\mathcal{P},u_i(P_i,\mathbf{P}_{-i})\rangle$, defined as follows. Let the set of allowed (power) strategies $\mathbf{P}$ be $\mathcal{P}=\{\mathbf{P}=[P_i]_{i\in\mathcal{S}}\colon 0\le P_i\le P_{\max}\}$, the utility function $u_i(P_i,\mathbf{P}_{-i})$ of the ith secondary node (player) in the subset $\mathcal{S}$ is defined as the difference between the achievable transmission rate and the cost of transmitted energy, similarly to, e.g., [19]. In particular, the rate achievable on the link between ST_i and SR_i is $\alpha(1-\beta)R_i(P_i,\mathbf{P}_{-i})$ with

$$R_i(P_i,\mathbf{P}_{-i}) = \log_2\left(1+\frac{|h_{\mathrm{S},ii}|^2 P_i}{N_0+\sum_{j=1 j\neq i}^{k}|h_{\mathrm{S},ij}|^2 P_j}\right), \quad (7)$$

where vector $\mathbf{P}_{-i}$ contains all the elements in $\mathbf{P}$ except the ith (i.e., it denotes the set of other players' strategies). Moreover, the cost for transmitted energy is $c\cdot\alpha P_i$ (recall that α is the overall fraction of time where the active secondary nodes are transmitting), with c being the cost per unit transmission energy[5]. Noticing that α is a common multiplier for both rate and energy cost and thus immaterial for the optimization of the utility, the utility function equivalently reads

$$u_i(P_i,\mathbf{P}_{-i}) = (1-\beta)R_i(P_i,\mathbf{P}_{-i}) - c\cdot P_i, \quad (8)$$

which depends on $\mathcal{S}$ and β, but is independent of α. We can conclude that the NE $\hat{\mathbf{P}}$ depends only on $\mathcal{S}$ and β and so does the rate R_{SP} in (4): this will be made clear by using the notation $\hat{\mathbf{P}}(\beta,\mathcal{S})$ for NE and $R_{\mathrm{SP}}(\beta,\mathcal{S})$ for (4).

C. Analysis

In this section, we provide some insight into the performance of cooperation-based spectrum leasing under the assumption of instantaneous CSI through analysis of the Stackelberg game at hand. In particular, we are interested in determining the conditions under which it is advantageous for the primary to lease the bandwidth for a fraction of time $\alpha>0$ to the secondary ad hoc network. We consider at first the non-cooperative power control game discussed in the previous section. It is well known that a NE is a fixed point of the best responses of all the nodes in $\mathcal{S}$ [12]. The best response of each user is obtained by writing the KKT conditions corresponding to the problem of maximizing the utility (8) with the respect to power P_i under the constraint $0\le P_i\le P_{\max}$ [13].

Proposition 1: The power control game $\langle\mathcal{S},\mathcal{P},u_i(P_i,\mathbf{P}_{-i})\rangle$ has always a NE. Moreover, a necessary condition for the NE to be unique is

$$\sum_{j\in\mathcal{S},\,j\neq i}\frac{|h_{\mathrm{S},ij}|^2}{|h_{\mathrm{S},ii}|^2} < 1. \quad (9)$$

Proof: It amounts to proving that the system of KKT conditions mentioned above has a unique solution, see [13] or [14].

The condition (9) for uniqueness of the NE is intuitive since it simply imposes an upper bound on the interference: in fact, with negligible interference, transmissions on different links ST_i-SR_i are uncoupled and the optimization of each utility (8) versus power P_i has a unique solution due to strict concavity.

We now turn to the performance of the overall Stackelberg game.

Proposition 2: The optimal fraction of time $\hat{\alpha}$ leased for secondary transmission and cooperation is strictly positive if

[4] We also recall that achieving rate (4) requires time synchronization at the symbol level among the secondary nodes, but does not require carrier synchronization (this is consistent with our assumption of lack of transmit CSI at the secondary nodes).

[5] Referring to the discussion in Sec. II-B, constant c can also account for the cost of the power spent for cooperation in case the latter is a linear function of the power used for secondary transmission.

and only if there exists a subset of secondary nodes $\mathcal{S} \subseteq \mathcal{S}_{tot}$ such that the following condition is satisfied

$$\frac{\hat{\beta} R_{\mathrm{SP}}(\hat{\beta}, \mathcal{S}) \cdot R_{\mathrm{PS}}(\mathcal{S})}{\hat{\beta} R_{\mathrm{SP}}(\hat{\beta}, \mathcal{S}) + R_{\mathrm{PS}}(\mathcal{S})} > R_{\mathrm{dir}} \tag{10}$$

where the optimal $\hat{\beta}$ is the solution of the optimization problem

$$\hat{\beta} = \arg \max_{\beta \in [0,1]} \beta R_{\mathrm{SP}}(\beta, \mathcal{S}). \tag{11}$$

Moreover, conditioned on (10) (i.e., on $\hat{\alpha} > 0$), the optimal fraction $\hat{\alpha}$ for a given subset $\mathcal{S}$ reads

$$\hat{\alpha}_{\mathrm{coop}} = \frac{1}{1 + \frac{\hat{\beta} R_{\mathrm{SP}}(\hat{\beta}, \mathcal{S})}{R_{\mathrm{PS}}(\mathcal{S})}}. \tag{12}$$

Proof: See Appendix-A.

The condition (10) provided by Proposition 2 simply states that cooperation is beneficial, and thus $\hat{\alpha} > 0$, if there exists a subset of secondary transmitters such that the cooperative primary rate R_{coop} (2) is larger than the direct rate R_{dir}. In fact, the left-hand side in (10) is the cooperative primary rate $R_{\mathrm{coop}}(\hat{\alpha}_{\mathrm{coop}}, \hat{\beta}, \mathcal{S})$ (2) with optimized $\hat{\alpha}_{\mathrm{coop}}$ (12) (see Appendix-A for details). This result can be summarized as follows: for any subset $\mathcal{S}$, the optimal primary rate is

$$R_{\mathrm{P}}(\hat{\alpha}, \hat{\beta}, \mathcal{S}) = \begin{cases} R_{\mathrm{coop}}(\hat{\alpha}_{\mathrm{coop}}, \hat{\beta}, \mathcal{S}) & \text{if (10) is satisfied} \\ R_{\mathrm{dir}} & \text{otherwise} \end{cases}. \tag{13}$$

Optimization (6) then requires in principle to consider all the 2^K subsets $\mathcal{S} \subseteq \mathcal{S}_{tot}$ for evaluation of (13). However, we will see in Sec. III-D that near-optimal results can be obtained with linear complexity in K. It is also noted that optimizing over β according to (11) requires a one-dimensional search (the problem is in general non-convex). Furthermore, from (12), it is interesting to notice that the optimal $\hat{\alpha}$, in case cooperation is employed (i.e., $\hat{\alpha}_{\mathrm{coop}}$), is chosen so as to avoid performance bottlenecks: it decreases (more time to primary transmission) if the channels from primary to secondary transmitters are the limiting factor (i.e., for increasing $\hat{\beta} R_{\mathrm{SP}}(\hat{\beta}, k)/R_{\mathrm{PS}}(\mathcal{S})$, while it increases (more time to secondary transmission) if the cooperative rate limits the overall performance (i.e., for decreasing $\hat{\beta} R_{\mathrm{SP}}(\hat{\beta}, k)/R_{\mathrm{PS}}(\mathcal{S})$).

D. Numerical examples

In this section, we consider a simple geometrical model where the set of secondary nodes are all placed at approximately the same normalized distance $0 < d < 1$ from the primary transmitter PT and $1 - d$ from the primary receiver PR[6]. Consequently, considering a path loss model (shadowing is not modelled), the average power gains of the channels read: $g_{\mathrm{P}} = 1$, $g_{\mathrm{PS},i} = 1/d^{\eta}$, and $g_{\mathrm{SP},i} = 1/(1-d)^{\eta}$, where $\eta = 2$ is the path loss coefficient. Moreover, in order to further reduce the number of system parameters and get better insight into the overall performance, we set $g_{\mathrm{S},ij} = \tilde{g}_{\mathrm{S}} = 1$ and $g_{\mathrm{S},ii} = g_{\mathrm{S}}$ for $i, j = 1, ..., K$ and $i \neq j$. The primary power and the maximum secondary transmission power are $P_{\mathrm{P}} = P_{\max} = 1$, the signal-to-noise ratio (SNR) is $SNR = P_{\mathrm{P}}/N_0 = 0\mathrm{dB}$, the cost per unit energy is $c = 0.1$ and, unless explicitly stated otherwise, the number of secondary transmitters is $K = 5$ and $g_{\mathrm{S}} = 10\mathrm{dB}$.

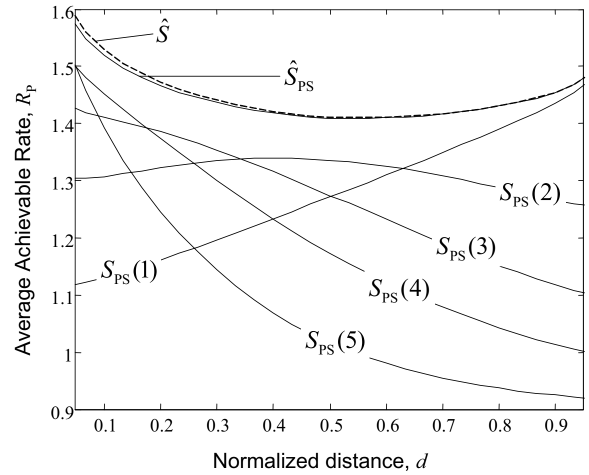


Fig. 2. Primary rate $R_{\mathrm{P}}(\hat{\alpha}, \hat{\beta}, \mathcal{S}_{PS}(k))$ (5), averaged over fading, versus the normalized distance d, for given subsets of secondary users $\mathcal{S}_{PS}(k)$, that contain the secondary users with the k best power gains from the primary user. Also shown is the comparison between the optimal rate $R_{\mathrm{P}}(\hat{\alpha}, \hat{\beta}, \widehat{\mathcal{S}})$ and the rate $R_{\mathrm{P}}(\hat{\alpha}, \hat{\beta}, \widehat{\mathcal{S}}_{PS})$ obtained by restricting the search to subsets $\mathcal{S}_{PS}(k)$ ($P_{\mathrm{P}} = P_{\max} = 1$, $SNR = 0\mathrm{dB}$, $K = 5$, $g_{\mathrm{S}} = 10\mathrm{dB}$).

In keeping with the description of the optimization procedure given in the previous section, Fig. 2 shows the optimal primary rate $R_{\mathrm{P}}(\hat{\alpha}, \hat{\beta}, \mathcal{S})$ (13), averaged via Monte Carlo simulations over the Rayleigh fading realizations, for given subsets of secondary users $\mathcal{S}$ and versus the normalized distance d. In particular, for reasons that will become apparent below, we focus on subsets $\mathcal{S}_{PS}(k) \subseteq \mathcal{S}$ which contain the secondary users ST_i with the k largest channel gains $|h_{\mathrm{PS},i}|^2$ from the primary transmitter (for each fading realization). It can be seen that for a secondary network placed at small distances it is better to activate (and thus cooperate with) a large number of secondary users given the large channel power gain from source to secondary network. Conversely, for large distances it is more convenient to cooperate only with the secondary users with the best instantaneous channel $|h_{\mathrm{PS},i}|^2$, exploiting multiuser diversity. The figure also compares the optimal primary rate $R_{\mathrm{P}}(\hat{\alpha}, \hat{\beta}, \widehat{\mathcal{S}})$ achieved via exhaustive search over the 2^K subsets $\mathcal{S}$ and the rate obtained by restricting the search only to the K subsets $\mathcal{S}_{PS}(k)$ with $k = 1, ..., K$. As it is clear, the reduction in order of complexity from 2^K to K entails almost no performance degradation[7].

Fig. 3 shows the optimal parameters $\hat{\alpha}$ and $\hat{\beta}$, averaged over fading distribution, versus the normalized distance d for the same setting as in Fig. 2. As it can be seen, the optimal value $\hat{\alpha}$ tends to decrease with distance d since, with increasing

[6]This assumption is mainly made in order to reduce the number of model parameters. Moreover, it should be noticed that the only secondary nodes that are eligible to be selected by the primary user are the ones that can enhance the primary throughput, which requires them to be located in between primary transmitter and primary receiver.

[7]It is also interesting to remark that, by focusing only on subsets of users of the class $\mathcal{S}_{PS}(k)$ (with $k = 1, ..., K$), the protocol can be simplified in that notification of the selected secondary nodes $\mathcal{S}_{PS}(k)$ can be made implicitly without exploiting control channels by simply selecting the transmission rate: only secondary nodes that are able to decode are to be considered as active (notice that this requires all the secondary nodes to attempt decoding).

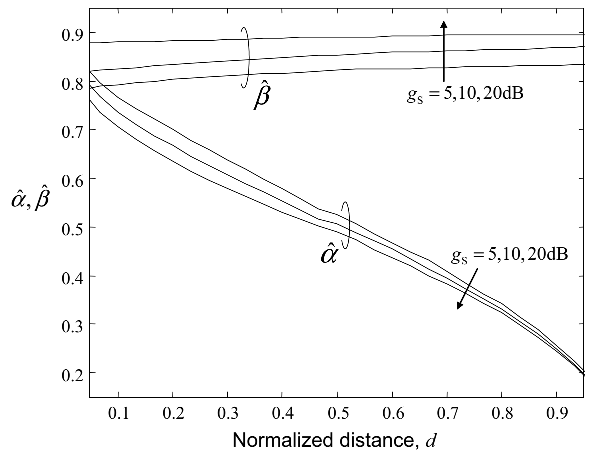


Fig. 3. Optimal value of parameters α and β, averaged over fading, versus the normalized distance d ($P_P = P_{\max} = 1$, $SNR = 0$dB, $K = 5$, $g_S = 10$dB).

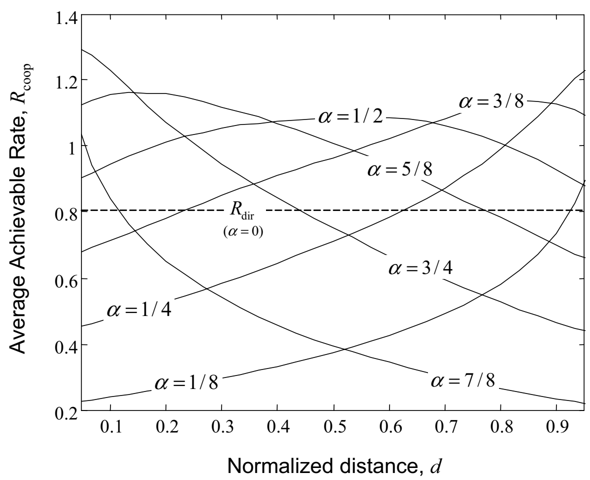


Fig. 4. Primary cooperative rate $R_{\text{coop}}(\alpha, \beta, \widehat{\mathcal{S}})$ (5), averaged over fading, versus the normalized distance d between the primary transmitters and primary receivers, for α ranging from $1/8$ to $7/8$, and $\beta = 0.8$. Dashed line refers to the rate achievable through direct transmission R_{dir} ($P_P = P_{\max} = 1$, $SNR = 0$dB, $K = 5$, $g_S = 10$dB).

d, activation of secondary nodes becomes more demanding (i.e., it requires more time), leaving less time for secondary transmission. This is in accordance to the discussion in the Sec. III-C around equation (12) (notice in fact that in this example $\hat{\alpha} = \hat{\alpha}_{\text{coop}}$ since $\hat{\alpha} > 0$). Moreover, it is seen that less interference between secondary channels (i.e., an increasing g_S) implies that the secondary nodes need to spend less power in order to optimize their utility (8), and therefore are willing to use less power for cooperation, which leads to smaller leased times α (and larger β).

In order to get further insight into the system behavior, Fig. 4 shows the cooperative rate $R_{\text{coop}}(\alpha, \beta, \widehat{\mathcal{S}})$ (2), averaged over different fading realizations, versus the normalized distance d, for α ranging from $1/8$ to $7/8$, with $\beta = 0.8$ and optimized $\mathcal{S}$. The rate on the direct link between PT and PR (1), R_{dir}, is also shown as a reference. As discussed above, as the distance d increases, the optimal $\hat{\alpha}$ decreases.

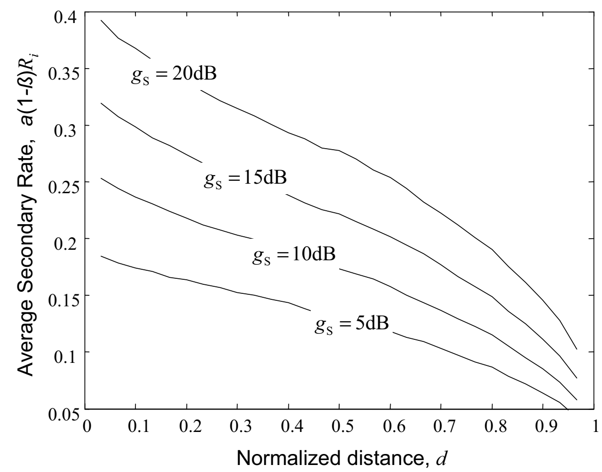


Fig. 5. Secondary rate $\alpha(1-\beta)R_i$ (7), averaged over fading, versus the normalized distance d, in a symmetric scenario with different channel power gains g_S between secondary transmitter-receiver pairs ($g_S = 5, 10, 15, 20$dB, $\tilde{g}_S = 0$dB, $P_P = P_{\max} = 1$, $SNR = 0$dB, $K = 5$).

Finally, the average rate achieved by the (activated) secondary user, $\alpha(1-\beta)R_i$, where R_i is given by (7), is shown in Fig. 5 as a function of the normalized distance d and the channel gain g_S. Since a larger distance entails a smaller optimal leased time α (see Fig. 3), the secondary rate decreases as the distance increases. Moreover, with increasing channel gain power g_S between secondary transmitter-receiver pairs (which entails better signal-to-interference ratios), as expected, the secondary nodes are able to achieve larger rates.

IV. Cooperation-based Spectrum Leasing with Long-term CSI and Randomized DSTC

In this section, we remove the assumptions of instantaneous CSI and ideal (information-theoretic) transmission schemes with block length $N_S \to \infty$ used in the first part of the paper. The unavailability of instantaneous CSI has several implications that motivate the system model considered in this section: (*i*) for any fixed transmission scheme (modulation and code) with rate $\bar{R}_P$ selected by the primary, there exist a non-zero probability of outage P_{out} due to possible (and unknown) adverse fading conditions; (*ii*) System optimization can only be based on probabilistic measures: for instance, the primary is not able to choose the subset $\mathcal{S}$ of secondary transmitters that decode and relay its message for a given channel realization, since secondary decoding is only guaranteed up to a given probability. This latter point makes the use of randomized DSTC techniques especially suitable for the considered application, as explained below.

In presence of instantaneous CSI, the primary directly informs each secondary transmitter belonging to the subset $\mathcal{S}$ of selected nodes about the space-time codeword to be transmitted in the second slot (recall Fig. 1). On the contrary, with long-term CSI, the set of secondary transmitters able to decode the primary message is a random quantity, and thus the choice of a space-time codebook and of the specific codeword to be transmitted by each node in $\mathcal{S}$ cannot be done by the primary. Therefore, randomized DSTC is a viable solution

code	q	L	R_{STC}
C_1	2	2	1
C_2	4	3	3/4
C_3	8	3,4	3/4
C_4	15	5	2/3
C_5	30	5,6	2/3
C_6	56	7	5/8

TABLE I
EXAMPLES OF A SET $\mathcal{C}$ OF ORTHOGNAL SPACE-TIME CODES WITH DIFFERENT SPATIAL DIMENSION L, TEMPORAL DIMENSION q AND RATE $R_{\rm ST}$ [22].

[17]. Randomized DSTC prescribes that each active secondary node selects *randomly* and *independently* a codeword within a given orthogonal space-time codebook $\mathbf{C}_i$. An orthogonal space-time codebook $\mathbf{C}_i$ is characterized by a number of codewords L_i (spatial dimension), number of symbols q_i (temporal dimension) and rate $R_{\rm STC,\it i} \leq 1$ (in information symbols per transmitted symbol). Therefore, the primary link only needs to inform the secondary nodes about the codebook $\mathbf{C}_i \in \mathcal{C}$ to be employed within a given set $\mathcal{C}$ of pre-defined options. An example of codebook set $\mathcal{C}$ is shown in Table I. We remark that this information needs to be communicated to all the secondary nodes only when the statistics of the channels have sufficiently changed, which is expected to happen after a large number of transmission slots.

In the rest of this section, we first discuss the Stackelberg model of interaction between the primary link (leader) and competitive secondary network (follower) in Sec. IV-A and Sec. IV-B, then numerical results are presented in Sec. IV-C in order to corroborate the analysis.

A. Primary link

The primary link wishes to transmit at a given target rate $\bar{R}_{\rm P}$ with a required bit error probability (BER). If the instantaneous channel fading conditions are such that the requirement on the BER is not satisfied, an outage is declared. The goal of the primary link is to minimize the probability of outage $\mathrm{P}_{out}(\alpha, \beta, \mathbf{C}_i)$ for fixed $\bar{R}_{\rm P}$ and BER with respect to α, β and the space-time codebook $\mathbf{C}_i \in \mathcal{C}$ (recall the example in Table I). Notice that the durations of different subslots are constrained to be integer numbers, so that the number of symbols $\alpha\beta N_{\rm S}$ in the second slot and the temporal dimension q_i of the codebook $\mathbf{C}_i$ needs to be related so that $\alpha\beta N_{\rm S}$ is an integer multiple of q_i.

If $\alpha > 0$, according to the employed multihop protocol, the primary destination decodes based on the signal received from the secondary network in the second slot (Fig. 1). Given the transmission scheme (herein considered as fixed), there exists a minimum SNR, $\gamma_{th}(R)$, that guarantees the desired BER for a given transmission rate R. This minimum SNR $\gamma_{th}(R)$ can be in general written as $\gamma_{th}(R) = (2^R - 1)/\Gamma_{\rm P}$, where parameter $\Gamma_{\rm P}$ is dependent on the transmission scheme and the target BER of the primary and is usually referred to as the gap parameter [20]. The outage probability $\mathrm{P}_{out}(\alpha, \beta, \mathbf{C}_i)$ is then the probability that the SNR on the cooperative link, here denoted as $\gamma_{\rm SP}(\mathcal{S}, \beta, \mathbf{C}_i)$ (see Appendix-B for further discussion), experiences an instantaneous SNR smaller than the desired threshold. This can be written as:

$$\mathrm{P}_{out}(\alpha, \beta, \mathbf{C}_i) = \sum_{\mathcal{S} \subseteq \mathcal{S}_{tot}} p_{\rm PS}(\mathcal{S}, \alpha) \cdot \mathrm{P}_{out,\rm SP}(\mathcal{S}, \alpha, \beta, \mathbf{C}_i), \tag{14}$$

where $p_{\rm PS}(\mathcal{S}, \alpha)$ is the probability that the secondary nodes in $\mathcal{S}$ are able to decode the primary transmission in the first slot and

$$\mathrm{P}_{out,\rm SP}(\mathcal{S}, \alpha, \beta, \mathbf{C}_i) = \Pr\left[\gamma_{\rm SP}(\mathcal{S}, \beta, \mathbf{C}_i) < \gamma_{th}\left(\frac{\bar{R}_{\rm P}}{\alpha\beta R_{{\rm STC},i}}\right)\right] \tag{15}$$

is the outage probability of the randomized DSTC in the second slot when nodes in $\mathcal{S}$ are active and the orthogonal STC $\mathbf{C}_i$ is used. Notice that transmission rate in the second slot is $\bar{R}_{\rm P}/(R_{{\rm STC},i} \cdot \alpha\beta)$ accounts for the duration of the second slot and the reduced rate $R_{{\rm STC},i} \leq 1$ of orthogonal STCs to attain an overall transmission rate $\bar{R}_{\rm P}$. Moreover, as detailed in Sec. IV-B, the SNR $\gamma_{\rm SP}(\mathcal{S}, \beta, \mathbf{C}_i)$ depends on the power selected by the secondary nodes, that is the outcome of a non-cooperative power game based on long-term CSI. Finally, if the leased time is $\alpha = 0$, we set the outage probability to $\mathrm{P}_{out}(0, \beta, \mathbf{C}_i) = \mathrm{P}_{out,\rm dir}$, where $\mathrm{P}_{out,\rm dir}$ is the outage probability on the direct primary channel: $\mathrm{P}_{out,\rm dir} = \Pr[P_{\rm P}\,|h_{\rm P}|^2 < \gamma_{th}(\bar{R}_{\rm P})] = [1 - \exp(-\gamma_{th}\left(\bar{R}_{\rm P}\right)/(P_{\rm P} \cdot g_{\rm P}))]$.

Calculation of $\mathrm{P}_{out,\rm SP}(\mathcal{S}, \alpha, \beta, \mathbf{C}_i)$ is discussed in Appendix-B based on the analysis in [21]. Furthermore, in order to simplify the evaluation of probabilities $p_{\rm PS}(\mathcal{S}, \alpha)$, here we assume that the average channel power gains from the primary transmitter to the secondary nodes have negligible differences (e.g., assuming a path loss model, the secondary terminals have essentially the same distance from the primary as in Sec. III-D) so that we can set $g_{{\rm PS},j} = g_{\rm PS}$ for $j = 1, ..., K$, and $p_{\rm PS}(\mathcal{S}, \alpha)$ is easily shown to become a binomial probability mass function with parameters k (number of nodes) and probability $q(\alpha) = \exp(-\gamma_{th}\left(\bar{R}_{\rm P}/(1-\alpha)\right)/(P_{\rm P} \cdot g_{\rm PS}))$.

To summarize, the primary link aims at solving the following optimization problem:

$$\begin{array}{l} \min\limits_{\alpha,\beta,\mathbf{C}_i} \ \mathrm{P}_{out}(\alpha, \beta, \mathbf{C}_i) \\ \text{s.t.} \left\{ \begin{array}{l} \mathbf{C}_i \in \mathcal{C},\ 0 \leq \alpha, \beta \leq 1, \\ \alpha N_S,\ \alpha\beta N_S,\ \alpha\beta N_S/q_i \in \mathcal{N} \end{array} \right. \end{array}, \tag{16}$$

where the outage probability of the primary link, $\mathrm{P}_{out}(\alpha, \beta, \mathbf{C}_i)$, is

$$\mathrm{P}_{out}(\alpha, \beta, \mathbf{C}_i) = \left\{ \begin{array}{ll} \mathrm{P}_{out,\rm dir} & \text{for } \alpha = 0 \\ \mathrm{P}_{out,\rm SP}(\mathcal{S}, \alpha, \beta, \mathbf{C}_i) & \text{for } \alpha > 0 \end{array} \right., \tag{17}$$

and $\mathcal{N}$ is the set of integers. The same interpretation in terms of Stackelberg games can be given as in Sec. III-A. Moreover, if the considered block length N_S is limited, optimization (16) can be reasonably dealt with through an exhaustive search over the optimization domain and without the aid of any sophisticated numerical algorithm. In fact, notice that: (*i*) the maximum number of STC orthogonal designs is strictly upper bounded by K; (*ii*) by considering the conditions αN_S, $\alpha\beta N_S \in \mathcal{N}$, one can show that the cardinality of the set of feasible α and β is upper bounded by $N_S(N_S - 1)/2$ (the third condition $\alpha\beta N_S/q_i$ can only reduce the feasible set). It

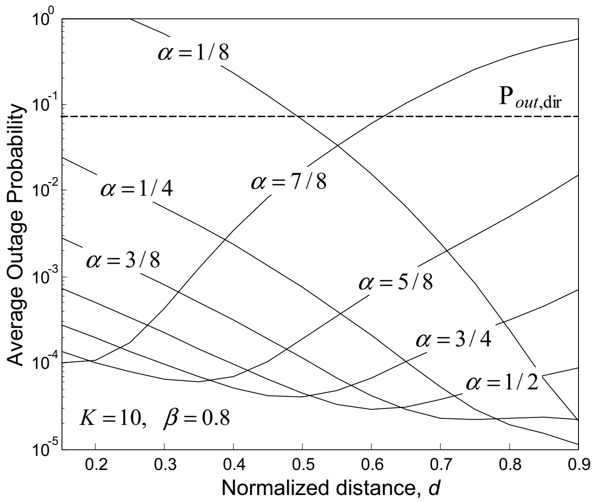


Fig. 6. Primary outage probability (14) versus the normalized distance d, for α ranging from $1/8$ to $7/8$, with $\beta = 0.8$ and orthogonal space-time code $\mathbf{C}_3$ from table I ($K = 10$ and $P_{\mathrm{P}} = P_{\max} = 2$, $SNR = 3dB$). Dashed line refers to the outage probabilty for direct transmission $\mathrm{P}_{out,\mathrm{dir}}$.

follows that the number of alternative solutions to be checked is strictly upper bounded by $KN_S(N_S - 1)/2$.

B. Ad hoc secondary network

As in Sec. III, the power selected by the secondary users in the activated subset $\mathcal{S}$ is the outcome of a non-cooperative power game. However, under the assumption of long-term CSI, the model considered in Sec. III-B is no longer applicable. Here we focus on a scenario where nodes in the secondary networks are assumed to be within short-range transmission with each other so that channels $h_{\mathrm{S},ij}$ contain a predominant line-of-sight component and $|h_{\mathrm{S},ij}|^2 = g_{\mathrm{S},ij}$. In this case, the game presented in Sec. III-B can be still applied even with long-term CSI since the utility (8) only depends on the deterministic channel gains $g_{\mathrm{S},ij}$, that can be easily estimated by the secondary nodes. Notice that in order to consider practical modulation schemes, the secondary rate (7) needs to be modified by including the gap parameter Γ_{S} that depends on the transmission mode of secondary transmitters as $R_i(P_i, \mathbf{P}_{-i}) = \log_2(1 + \Gamma_{\mathrm{S}} |h_{\mathrm{S},ii}|^2 P_i/(N_0 + \sum_{j=1 j\neq i}^{k} |h_{\mathrm{S},ij}|^2 P_j))$. We refer to this condition as *disk model* in order to emphasize that the secondary terminals are within direct transmission range from one another. An alternative model suitable for a scenario with fading attenuations in the secondary network can prescribe utility functions that depend on the secondary outage probability, see, e.g., [15].

C. Numerical examples

Here we discuss some numerical results so as to provide insight into the advantages of spectrum leasing via cooperation in presence of long-term CSI. We employ the same geometry-based model used in Sec. III-D and assume the disk model for the secondary network (see Sec. IV-B). Moreover, where not stated otherwise, we set the power gains in the secondary network to $\tilde{g}_S = |h_{\mathrm{S},ij}|^2 = 1$ and $g_{\mathrm{S}} = |h_{\mathrm{S},ii}|^2 = 20$dB. Other parameters are selected as $P_{\mathrm{P}} = P_{\max}$ (where not explicitly stated), $N_0 = 1$, $\bar{R}_P = 0.3$, number of symbols per slot $N_{\mathrm{S}} = 80$ and energy cost $c = 0.1$. The gap parameters Γ_{P} and Γ_{S} read $\Gamma_{\mathrm{P}} = \Gamma_{\mathrm{S}} = -1.5/\log(5 \cdot b)$ [20], where $b = 10^{-3}$ is the required BER.

Fig. 6 shows the primary outage probability P_{out} (14) versus the normalized distance d, for different values of α, with $\beta = 0.8$, STC $\mathbf{C}_3$ in Table I, $SNR = P_{\mathrm{P}}/N_0 = 3$dB and $K = 10$ secondary nodes. The outage probability for direct transmission $\mathrm{P}_{out,\mathrm{dir}}$ (dashed line) is also shown as references. It can be seen that significant outage improvements with respect to direct transmission can be achieved by appropriately choosing α, and that the optimal fraction of leased time α tends to decrease for increasing distance d. It is interesting to notice that this is the same behavior as in the case of instantaneous CSI depicted in fig. **??**.

The primary outage probability is then shown in Fig. 7 for optimized system parameters α, β and $\mathbf{C}_i$ according to (16) versus the normalized distance d. Different values of the number of nodes in the secondary network K are considered, $P_{\mathrm{P}} = P_{\max}$ (thus $SNR = 3$dB) on the left and $P_{\mathrm{P}} = 2P_{\max}$ (thus $SNR = 6$dB) on the right. For each curve, dashed lines specify boundaries of the areas where the same complex orthogonal design $\mathbf{C}_i$ is used. First-order discontinuities in the boundary regions are due to the different diversity orders provided by different STCs. In particular, for decreasing distance d (secondary network approaching the primary transmitter) the number of decoding nodes k increases and codebooks with larger number of codewords L (and thus larger diversity) can be selected. Moreover, for a fixed code $\mathbf{C}_i$, the outage probability at first decreases with decreasing d due to the larger number of decoding secondary nodes, but eventually increases when too many nodes are active and tend to select the same codeword with large probability (see [17] [21]). As a final remark, notice that when the SNR and the number of users K are large enough ($SNR = 6$dB and $K > 13$), the minimum outage is achieved when the secondary network is closer to the receiver since with large primary power secondary users are easily activated.

To give further insight into the optimal choice of parameters α and β, here we provide a numerical example referred to the setting of Fig. 7. For $K = 10$, when the secondary nodes are close to the primary ($d = 0.3$), the optimal leased time is large ($\hat{\alpha} \simeq 3/4$ and $\hat{\beta} \simeq 3/4$), whereas when the secondary nodes are far from the primary ($d = 0.7$), the optimum leased time is smaller ($\hat{\alpha} \simeq 2/5$, and $\hat{\beta} \simeq 3/4$). As noted above, this is the same qualitative behavior that hold in the case of instantaneous CSI, as it can be seen from fig. 3.

V. Concluding remarks

In this paper, we have proposed and investigated a solution for spectrum leasing based on the idea that secondary nodes can earn spectrum access in exchange for cooperation with the primary link. The analysis is meant to shed light on possible approaches for the implementation of the property rights model of cognitive radio based on the current wireless technology. The basic premise of the proposal is indeed that spectrum leasing may take place not necessarily on the basis

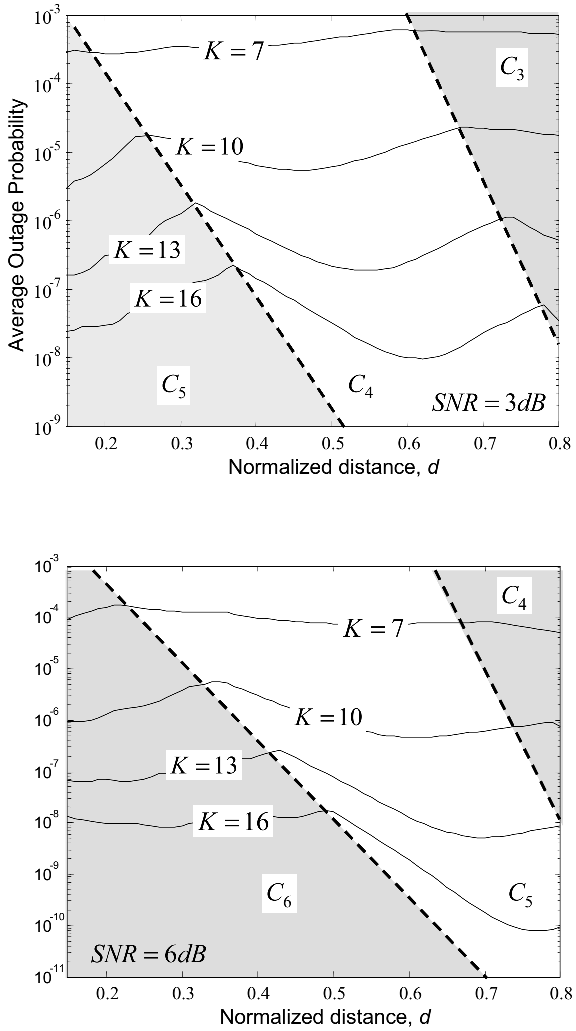


Fig. 7. Primary outage probability with optimal α, β and $\mathbf{C}_i \in \mathcal{C}$ in table I according to the optimization problem (16) versus the normalized distance d for $K = 7, 10, 13, 16$ secondary nodes, $P_P = P_{\max}$ ($SNR = 3dB$ on the top) and $P_P = 2P_{\max}$ ($SNR = 6dB$ on the bottom).

of fees or charges, but also in return for an improved quality-of-service of the incumbent primary users via cooperation with secondary terminals. By casting the problem in the framework of Stackelberg games, we have provided analytical and numerical results that have confirmed the considered model as a promising paradigm for cognitive radio networks.

VI. Appendix

A. Appendix-A: Proof of Proposition 2

Recalling that the NE does not depend on α but only on $\mathcal{S}$ and β, the cooperative primary rate (2) can be conveniently restated as $R_{\mathrm{coop}}(\alpha, \beta, \mathcal{S}) = \min\{(1-\alpha)\, R_{\mathrm{PS}}(\mathcal{S}), \alpha\beta R_{\mathrm{SP}}(\beta, \mathcal{S})\}$. Parameter β only appears in the second term of (5) and therefore can be optimized independently by solving problem (11) for a given choice of $\mathcal{S}$. Moreover, it is easy to observe that $R_{\mathrm{coop}}(\alpha, \beta, \mathcal{S})$ is the minimum of a decreasing function of α, $(1-\alpha)\, R_{\mathrm{PS}}(\mathcal{S})$, and an increasing function of α, $\alpha\beta R_{\mathrm{SP}}(\beta, \mathcal{S})$, and therefore maximization is achieved when the two terms are equal: $(1-\hat{\alpha}_{\mathrm{coop}})\, R_{\mathrm{PS}}(\mathcal{S}) = \hat{\alpha}_{\mathrm{coop}}\hat{\beta} R_{\mathrm{SP}}(\hat{\beta}, \mathcal{S})$. This condition results in (12). Finally, plugging the optimal $\hat{\alpha}_{\mathrm{coop}}$ (12) in $R_{\mathrm{coop}}(\hat{\alpha}_{\mathrm{coop}}, \hat{\beta}, \mathcal{S})$, we get that the maximum primary rate achievable with cooperation reads

$$R_{\mathrm{coop}}(\hat{\alpha}_{\mathrm{coop}}, \hat{\beta}, \mathcal{S}) = \frac{\hat{\beta} R_{\mathrm{SP}}(\hat{\beta}, \mathcal{S}) \cdot R_{\mathrm{PS}}(\mathcal{S})}{\hat{\beta} R_{\mathrm{SP}}(\hat{\beta}, \mathcal{S}) + R_{\mathrm{PS}}(\mathcal{S})}, \quad (18)$$

from which condition (10) easily follows.

B. Appendix-B: Evaluation of $\mathrm{P}_{out,\mathrm{SP}}(\mathcal{S}, \alpha, \beta, \mathbf{C}_i)$ in (15)

Given a codebook $\mathbf{C}_i$ and a subset of $\mathcal{S}$ of active secondary transmitters, the performance of the DSTC scheme depend essentially on the probability that more terminals select the same space-time codeword matrix row, thus yielding a loss of diversity. In order to simplify the analysis, here we focus on the disk model and assume that the secondary network is symmetric, i.e., the channels satisfy the conditions $|h_{\mathrm{S},ii}|^2 = g_{\mathrm{S}}$ and $|h_{\mathrm{S},ij}|^2 = \tilde{g}_{\mathrm{S}}$ for $i, j = 1, ..., K$ and $i \neq j$. Under this assumptions, one can show that, by symmetry, the NE of the power game within the secondary networks is characterized by equal powers $\hat{P}_i(\beta, \mathcal{S})$, which can be calculated as $\hat{P}(\beta, \mathcal{S}) = [((1-\beta)g_S/c - N_0)(1/(g_S - \tilde{g}_S + k\tilde{g}_S))]_0^{P_{\max}}$ and depends only on the cardinality k of $\mathcal{S}$ (see discussion in Sec. III-B). Assuming the further condition that the average channel gains from secondary nodes to primary receiver are equal (as in the geometric model considered in Sec. III-D and Sec. IV-C), we have $g_{\mathrm{SP},i} = g_{\mathrm{SP}}$ for $i = 1, ..., K$ and the outage performance of the randomized DSTC with given spatial dimension L and temporal dimension $q = \alpha\beta N_S \geq L$ can be obtained from [21]. In particular, reference [21] derives the equivalent SNR $\gamma_{\mathrm{SP}}(\mathcal{S}, \beta, \mathbf{C}_i)$ on the cooperative link and shows that in the high SNR regime ($\hat{P} \cdot g_{\mathrm{SP}} >> N_0$) the probability of outage reads

$$\mathrm{P}_{out,\mathrm{SP}}(\mathcal{S}, \alpha, \beta, \mathbf{C}_i) \simeq \frac{1}{L_{\mathrm{eff}}!} \left(\frac{\gamma_{th}\left(\frac{\bar{R}_{\mathrm{P}}}{\alpha\beta R_{\mathrm{STC},i}}\right)}{\bar{\rho}} \right)^{2^{-1/k} L_{\mathrm{eff}}} \cdot \left(1 + \frac{\bar{\rho}}{\bar{\rho}_t}\right)^{2^{-1/k} L_{\mathrm{eff}} - 1} \quad (19)$$

where $L_{\mathrm{eff}} = \min\{k, L_i\}$ is the maximum diversity of the code and $\bar{\rho} = k\hat{P}(\beta, k) g_{\mathrm{SP}}/(L_{\mathrm{eff}} N_0)$ is the average SNR over each diversity branch and $\bar{\rho}_t = \gamma_{th}\left(\frac{\bar{R}_{\mathrm{P}}}{R_{\mathrm{STC}} \cdot \alpha\beta}\right) L_{\mathrm{eff}}^{k-1/(L_{\mathrm{eff}}-1)}$.

References

[1] J. O. Neel, *Analysis and Design of Cognitive Radio Networks and Distributed Radio Resource Management Algorithms*, PhD. dissertation, Virginia Polytechnic Institute, Sept. 2006.

[2] C. Tawil and S. Tawil, "Apparatus and method for reusing satellite broadcast spectrum for terrestrially broadcast signals," Patent number: 6519446, Assignee: Northpoint Technology, Ltd, Issue date: Feb. 11, 2003.

[3] http://www.ieee802.org/21/

[4] C. Cordeiro, K. Challapali, D. Birru and Sai Shankar N, "IEEE 802.22: the first worldwide wireless standard based on cognitive radio," in *Proc. IEEE DySPAN*, pp. 328 - 337, 2005.

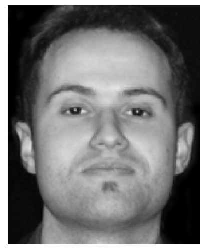
[5] J. M. Peha, "Approaches to spectrum sharing," *IEEE Commun. Mag.*, vol 43, no. 2, pp. 10-12, Feb. 2005.

[6] G. Faulhaber and D. Farber, "Spectrum management: Property rights, markets and the commons," in *Proc. Telecommunications Policy Research Conference*, Oct. 2003.

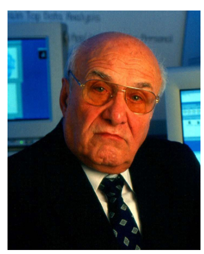
[7] N. Devroye, P. Mitran and V. Tarokh, "Achievable rates in cognitive radio," *IEEE Trans. Inform. Theory*, vol. 52, no. 5, pp. 1813-1827, May 2006.
[8] A. Jovicic and P. Viswanath, "Cognitive radio: an information-theoretic perspective," submitted.
[9] Y. Chen, Q. Zhao and A. Swami, "Joint design and separation principle for opportunistic spectrum access," in *Proc. Asilomar Conf. on Signals, Systems and Computers,* 2006.
[10] O. Simeone, Y. Bar-Ness and U. Spagnolini, "Stable throughput of cognitive radios with and without relaying capability," to appear in *IEEE Trans. Commun.*
[11] J. N. Laneman and G. W. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.
[12] M. J. Osborne and A. Rubenstein, *A Course in Game Theory*, MIT Press, 1994.
[13] G. Scutari, D. P. Palomar and S. Barbarossa, "Optimal Linear Precoding/Multiplexing for Wideband Multipoint-to-Multipoint Systems based on Game Theory-Part I: Nash Equilibria," to appear in *IEEE Trans. Signal Processing*.
[14] R. Etkin, A. Parekh and D. Tse, "Spectrum Sharing for Unlicensed Bands," in *Proc. Allerton Conference on Communication, Control and Computing, 2005*.
[15] O. Ileri, S. C. Mau and N. B. Mandayam, "Pricing for Enabling Forwarding in Self-Configuring Ad Hoc Networks," *IEEE J. Sel. Areas Commun.*, vol. 23, No. 1, pp. 151–162, Jan. 2005.
[16] I. Stanojev, O. Simeone and Y. Bar-Ness, "Optimal Design of a Multi-Antenna Access Point with Decentralized Power Control Using Game Theory," Proc. *IEEE DySPAN 2007*.
[17] A. Scaglione, Dl. L. Goeckel and J. N. Laneman, "Cooperative communications in mobile ad hoc networks," *IEEE Signal Processing Mag.*, vol. 23, no. 5, pp. 18-29, Sept. 2006.
[18] A. Høst-Madsen and J. Zhang, "Capacity Bounds and Power Allocation for Wireless Relay Channel," *IEEE Trans. Inform. Theory,* vol. 51, No. 6, pp. 2020-2040, June 2005.
[19] J. Huang, R. A. Berry and M. L. Honig, "Distributed Interference Compensation for Wireless Networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 5, pp. 1074-1084, May 2006.
[20] G. J. Foschini and J. Salz, "Digital communications over fading radio channels," *Bell Systems Tech. Journal*, pp. 429–456, 1983.
[21] S. Savazzi and U. Spagnolini, "Distributed orthogonal space-time coding: design and outage analysis for randomized cooperation," to appear in *IEEE Trans. Wireless Commun.*
[22] X.B. Liang, "Orthogonal designs with maximal rates," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2468 – 2503, Oct. 2003.
[23] A. Ghasemi and E. S. Sousa, "Collaborative Spectrum Sensing for Opportunistic Access in Fading Environments," in *Proc. IEEE DySPAN*, Nov. 2005.
[24] M. Sharif, B. Hassibi "On the capacity of MIMO broadcast channels with partial side information," *IEEE Trans. Inform. Theory*, vol. 51, no. 2, pp. 506-522, Feb. 2005.

Osvaldo Simeone (M02) received the M.Sc. degree (with honors) and the Ph.D. degree in Information Engineering from Politecnico di Milano, Milan, Italy, in 2001 and 2005 respectively. He is currently with the Center for Wireless Communications and Signal Processing Research (CWCSPR), at the New Jersey Institute of Technology (NJIT), Newark, New Jersey, where he is an Assistant Professor. His current research interests concern the cross-layer analysis and design of wireless networks with emphasis on information-theoretic, signal processing and queuing aspects. Specific topics of interest are: cognitive radio, cooperative communications, ad hoc, sensor, mesh and hybrid networks, distributed estimation and synchronization.

Igor Stanojev (S'03) received the Dipl.Ing. degree in electrical engineering from the University of Belgrade, Belgrade, Serbia, in 2001 and the M.S. degree in electrical engineering from New Jersey Institute of Technology (NJIT), Newark, NJ, in 2006, where he is currently pursuing the Ph.D. degree in the Center for Wireless Communications and Signal Processing (CWCSPR). His research interests are in the crosslayer design, including the cooperative communications, Hybrid ARQ protocols, and distributed networks.



Stefano Savazzi (S'05) received the M.Sc. degree (with honors) from the Politecnico di Milano, Milan, Italy, in December 2004, where he is currently working toward the Ph.D. degree. He was a Visiting Researcher at the Signals and Systems department, Uppsala University, Sweden, and at the Electrical and Computer Engineering, University of California and San Diego, US. He holds a patent on the work developed for his M.S. thesis. His current research interests include signal processing aspects for digital wireless communications and, more specifically, antenna array processing for MIMO systems and wireless relay channels.



Yeheskel Bar-Ness holds B.Sc and M.Sc degrees in Electrical Engineering from the Technion, Israel, and Ph.D. degree in Applied Mathematics from Brown University, Providence, RI. He is a Distinguished Professor of ECE, Foundation Chair of Communication and Signal Processing Research and Executive Director of the Center for Wireless Communication and Signal Processing Research (CCSPR) at the New Jersey Institute of Technology, Newark. After working in the private sector, he joined the School of Engineering, Tel-Aviv University in 1973. He came to NJIT from AT&T Bell Laboratories in 1985. Current research interests include design of MIMO-OFDM, and MC-CDMA, adaptive array and spatial interference cancellation and signal separation for multi-user communications, and modulation classification. Recently, he is contributing in the area of cooperative communication, modulation classification, cognitive radio, link adaptation with cooperative diversity, cross layer design and analysis and scheduling and beam-forming for downlink with limited feedback. He published numerous papers in these areas. He serves on the editorial board of WIRED Magazine, was the founding Editor-in-chief of IEEE Communication Letters, and was associate and area editor for IEEE Transactions on Communications. He is currently on the editorial board of the Journal of Communication Networks. He has been the technical chair of several major conferences and symposiums and was the recipient of the Kaplan Prize (1973), which is awarded annually by the government of Israel to the ten best technical contributors. He is a Fellow if IEEE and is the recipient of the IEEE Communication Societys Exemplary Service Award, and was selected the NJ 2006 Inventor of the Year, recognized for systems and methods to enhance wireless/mobile communications.

Umberto Spagnolini (SM03) received the Dott.Ing. Elettronica degree (cum laude) from Politenico di Milano, Milan, Italy, in 1988. Since 1988, he has been with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, where he is Full Professor in Telecommunications. His general interests are in the area of statistical signal processing. The specific areas of interest include channel estimation and space-time processing for wireless communication systems, parameter estimation and tracking, signal processing and wavefield interpolation applied to UWB radar, geophysics, and remote sensing. Dr. Spagnolini serves as an Associate Editor for the IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING.

Raymond L. Pickholtz , professor emeritus and former chairman of the Department of Electrical and Computer Engineering at The George Washington University received his Ph.D. in Electrical Engineering from the Polytechnic Institute of Brooklyn. He has worked on many aspects of wireless communications including Spread Spectrum, Cellular Systems, Ad-Hoc Networking, CDMA, Satellite Systems, Digital Audio Broadcasting and UWB. He has authored or co-authored several hundred technical papers. He has served on many government and industry advisory boards and has been a consultant to most of the major communications companies in the U.S. He served on the FCC working group of the Spectrum Policy Task Force, 2002. He has been an active member of the IEEE Communications Society where he served as an Editor, Chairman of Major Conferences, on the Board of Governors, and as President. He received the Donald W. McLellan Award in 1994. He was a visiting Erskine Fellow at the University of Canterbury, Christchurch, NZ, 1997. He was the General Chairman of (IEEE) Infocom97 in Kobe, Japan in 1997 and of (ACM) Mobicom2000 in Boston in 2000. He co-authored the ”Best paper” of 1999 in the Journal of Communications and networks. He was named Co Editor in Chief of that journal in 2002, and Editor in Chief in 2004. Dr Pickholtz was awarded the IEEE Millenium Medal in 2000. He is a fellow of the Institute of Electrical and Electronic Engineers (IEEE), of the Washington Academy of Science, and of the American Association for the Advancement of Science (AAAS).