

Cooperative Fading Regions for Decode and Forward Relaying

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Abstract—Cooperative transmission protocols over fading channels are based on a number of relaying nodes to form virtual multi-antenna transmissions. Diversity provided by these techniques has been widely analyzed for the Rayleigh fading case. However, short range or fixed wireless communications often experience propagation environments where the fading envelope distribution is meaningfully different from Rayleigh. The main focus in this paper is to investigate the impact of fading distribution on performances of collaborative communication. Cooperative protocols are compared to co-located multi-antenna systems by introducing the concept of cooperative fading region. This is the collection of fading distributions for which relayed transmission can be regarded as a competitive option (in terms of performances) compared to multi-antenna direct (noncooperative) transmission. The analysis is dealt with by adopting the information theoretic outage probability as the performance metric. Cooperative link performances at high SNR are conveniently expressed here in terms of diversity and coding gain as outage parameters that are provided by the fading statistics of the channels involved in collaborative transmission. Advantages of cooperative transmission compared to multi-antenna are related to the propagation environment so that the analysis can be used in network design.

Index Terms—Cooperative diversity, cooperative transmissions, decode and forward relaying, distributed space-time coding, fading channels.

I. INTRODUCTION

MULTIPATH fading is one of the main limitations in wireless ad hoc and cellular networks as it causes meaningful link performance degradation that is almost independent of the network size. Depending on the nature of the propagation environment, there are different models describing the statistical behavior of the multipath fading envelope and thus of the signal-to-noise power ratio (SNR). Although the Rayleigh distribution is widely used to model multipath fading with no direct line-of-sight (NLOS) path, ad-hoc, sensors networks and more generally fixed or short range applications, often experience propagation environments where the power of the line-of-sight

(LOS) component is either non zero or, in some cases, dominate when compared to the random NLOS components. Therefore, the influence of the propagation settings onto the overall system performance depends on the statistics of the random fading power, or better on the relevance of the diffusive (or NLOS) contribution with respect to the direct (or LOS) [1].

Multiple antenna transmissions are known to provide spatial redundancy (or diversity order) to reduce fading impairments. In particular, the use of space-time codes is shown to combat the fading effects by harnessing the diversity of the channel without channel state information at the transmitter. Since hardware, size and cost constraints limit the number of antennas that can be deployed, recently it has been shown that instead of a single terminal with multiple antennas, a cluster of single-antenna nodes might form a kind of coalition to cooperatively act as a transmit or receive array [2]. When terminals collaborate, they typically first exchange messages to set up the coordinated transmission and then cooperatively transmit the information symbols thus creating a distributed multi-antenna system. Performances can thus benefit from the cooperative diversity [3] provided by multiple input (and, in case, multiple output) virtual channel. Simple transmission protocols that can exploit cooperation were first investigated for the Rayleigh fading case in cellular [4] and ad hoc [2], [5] networks. In these works, transmissions from source and relay(s) towards the destination take place either through orthogonal channels (e.g., time, frequency or code division) or by varying the degree of receiver collisions [5]. Instead, in [3] *cooperative diversity* is assessed by employing multiple coordinated relays that share the same bandwidth to arrange a (virtual) antenna array by means of distributed space-time coding (D-ST). In [6] the analysis of cooperative systems is dealt with for the Rayleigh fading case by defining the best relay location for cooperation to be beneficial with respect to direct transmission. In this paper, the performance analysis of cooperative transmissions with decode and forward relaying is dealt with by embracing *arbitrary* fading distributions for each link that is scheduled for transmission. As a motivation to this work, consider the propagation setting in Fig. 1 where a source node S might decide to reach the destination D with the help of relayed transmissions from a cluster of N nodes. Different channel power distributions are experienced for the source-to-relays $f_{|h_{sr}|^2}(\cdot)$, relays-to-destination $f_{|h_{rd}|^2}(\cdot)$ and source-to-destination $f_{|h_{sd}|^2}(\cdot)$ links (e.g., notice that relays might be placed either in short range with the source, or in strategic locations to benefit from a marginal diffusive channel component). For a given propagation environment, the main focus in this paper is to investigate the necessary conditions on the channel power distributions of fading that makes

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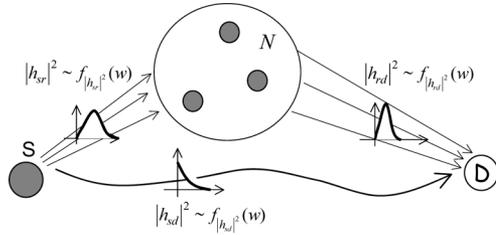


Fig. 1. Propagation environment for cooperative and noncooperative transmissions.

relayed transmissions to perform as if the source node would be equipped with multiple antennas and employ a direct (non-cooperative) multiple-input–single-output (MISO) transmission towards the destination. To get more insight into how the propagation settings can influence the performances of collaborative transmission, we introduce the concept of cooperative fading regions \mathcal{R} as the collection of channel power distributions that make cooperation preferable (or comparable) to conventional noncooperative MISO transmission according to a specific performance criteria.

A. Outage Probability Based Analysis

The information theoretic outage probability is the criterion employed in this paper to evaluate the system performances. The outage probability can be asymptotically (for large signal to noise ratio: SNR) approximated as $P_{\text{out}} \sim (c \times \text{SNR})^{-d}$, with the “inherent” diversity order d and coding gain c as two outage parameters that depend on the fading distribution of the channel itself [1]. The reference setting for performance comparison is based on a multiple antenna transmission from source S towards the destination D (Fig. 2(a)). Link performances are characterized by an inherent fractional diversity order d_{sd} and coding gain c_{sd} according to the statistical property of the channel. Cooperative transmission protocols are shown in Fig. 2(b)–(e). The source node S first activates N relaying nodes by using N links that have the same fading distribution and thus are characterized by the inherent diversity d_{sr} and coding gain c_{sr} . Relayed transmission towards the destination is performed over a channel that provides a diversity d_{rd} and a coding gain of c_{rd} . The pairs (d_{sd}, c_{sd}) , (d_{sr}, c_{sr}) and (d_{rd}, c_{rd}) are based on the corresponding fading distributions and can be interpreted as (outage) parameters that model the outage performance over each link (see Fig. 1). Differently from [1] and [7], here we show that these outage parameters can be evaluated in a general way from the asymptotic behavior of the moment generating function (MGF) of the random fading power over each link.

Decode and forward based collaborative protocols are analyzed in this paper by evaluating the outage parameters (i.e., the diversity and coding gain pairs for each link) that make the cooperation to be beneficial in improving the outage probability when compared to standard noncooperative MISO transmission (Fig. 2(a)). When the outage probability is used as performance metric, the cooperative fading regions turns out to be the collection of outage parameters based on the pairs (d_{sd}, c_{sd}) , (d_{sr}, c_{sr}) and (d_{rd}, c_{rd}) for which cooperation has better outage performances compared to conventional MISO transmission in Fig. 2(a) used as reference setting.

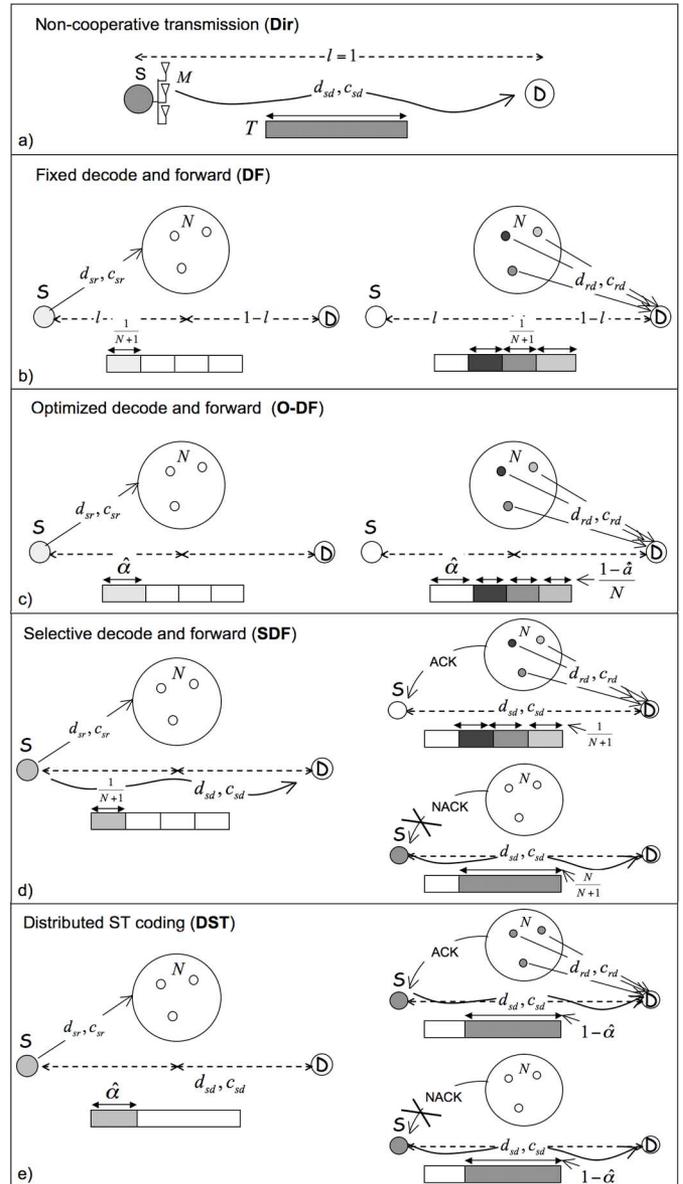


Fig. 2. (a) Direct noncooperative transmission, (b) DF, (c) O-DF, (d) S-DF, and (e) D-ST cooperative transmission protocols. Each link is characterized by the outage parameters as inherent diversity (d) and coding gain (c). For each protocol, shaded nodes refer to terminals that are scheduled for transmitting during a specific time fraction.

B. Overview of the Main Results

In what follows, we outline the original contributions of the paper as compared to the existing works.

- i) Section III extends the outage analysis of cooperative protocols for fixed and selective (or adaptive) decode and forward relaying methodologies [2], [3] (an overview of existing results is given in Section II) in order to embrace arbitrary fading statistics over each link. Fading channels are described by the inherent (fractional) diversity (d) and coding gain (c) parameters as in [1]. However, differently from [1] these outage parameters are evaluated in a general way from the asymptotic behavior of the moment generating function (MGF) of the random fading power over each link. Compared to

- [7], this novel approach to performance evaluation of wireless communications requires only the knowledge of the MGF of the measured fading power distribution. MGF is a standard concept either in communication field [22] and in mathematical statistics [18]. The proposed method (details are in Appendix A) allows for closed form high SNR performance evaluation of distributed MISO (or MIMO) systems under arbitrary propagation environments (that might include fractional diversities and unbalanced fading cases).
- ii) The concept of *cooperative diversity* [2] is generalized herein to embrace arbitrarily distributed fading channels. Cooperative diversity is shown to depend on both the fading statistics and on the spatial redundancy offered by the specific transmission methodology and coding algorithm. In Section IV, we define the asymptotic cooperative fading regions as the collections of fading power distributions that makes collaborative transmissions to provide a (cooperative) diversity that is larger than the diversity provided by noncooperative multi-antenna MISO transmissions. In Section VII, we show how cooperative diversity for arbitrary fading can impact on diversity-multiplexing tradeoff (DMT) analysis.
 - iii) Cooperative fading regions are defined in Section V by assuming (for both direct and collaborative transmissions) that only a finite amount of power is available. Regions highlight the necessary conditions on the channel power distributions of fading that make relayed transmissions to outperform noncooperative ST-coded multiple antenna transmission (here used as reference setting). With the aim of providing fading regions that can be readily used for practical network design, closed form bounding regions are also developed for each protocol by using the large SNR outage approximation (technicalities are confined in Appendix B). Bounding regions can be used either to highlight conditions (e.g., requirements for the fading distributions) that are *necessary* for cooperation to enhance (or to perform as) direct transmission, or rather more practical conditions that are *sufficient* to declare the cooperation as impracticable compared to co-located multi-antenna MISO transmission.

The rest of the paper is organized as follows. As a case study, in Section VI asymptotic and finite SNR cooperative fading regions are specialized to fit with Nakagami-m fading environments. Analysis of fading regions is dealt with numerically. Finally, we provide some basic design rules drawn from numerical analysis and valid for different applications and propagation settings.

II. OVERVIEW ON OUTAGE ANALYSIS OF COOPERATIVE TRANSMISSION PROTOCOLS

The following analysis focuses on different cooperative protocols that involve transmission from a single antenna source node S towards a single antenna destination node D (extension to multiple antennas at the receiver is straightforward) with the use of the channel for a time interval T (without any loss of generality, it can be set $T = 1$). Destination can be reached (as

in Figs. 1 and 2) by a cooperative transmission where source S activates N neighbor nodes to relay the message either by simply repeating (after decoding) or by engaging a virtual multi-antenna system through a distributed ST coding. The direct ST-coded transmission from a source node equipped with $M \geq 1$ antennas (Fig. 2(a)) represents the reference setting to be used for performance comparison of cooperative transmission protocols.

As a baseline case, let E_S be the transmitted symbol energy from a single antenna source towards a destination, the transmit power is $\rho = E_S/T$ and the signal to noise ratio (SNR) referred to the transmitting side is ρ/σ^2 , with $\sigma^2 = N_0/T$ the additive white Gaussian (AWGN) noise power and N_0 the single-sided noise power spectral density (to simplify, here we assume $N_0 = 1$ and $\sigma^2 = 1$ so that ρ refers equivalently to SNR or transmit power).

The baseband complex valued channel gain is h , fading power $E[|h|^2] = g$ accounts for path loss and shadowing, the instantaneous SNR at the receiving side is $\mu = \rho|h|^2$. Assuming a channel with static fading for the whole transmission duration, the maximum mutual information over the link for source employing Gaussian codebook is $I = \log_2(1 + \mu)$, for a target rate R [b/s/Hz], the outage probability is $\Pr[I < R] = \Pr[\mu < 2^R - 1]$.

In the direct link with M antennas (Fig. 2(a)) the diversity is exploited by an orthogonal ST coded transmission. The outage probability for noncooperative multiantenna transmission is [10]

$$\Pr[I_{sd}(M) < R] = \Pr[\mu_{sd}(M) < 2^R - 1], \quad (1)$$

where subscript sd refers to the source-to-destination direct link and $\mu_{sd}(M) = (\rho/M) \sum_{i=1}^M |h_{sd,i}|^2$. Notice that transmit power of each antenna scales as ρ/M in order to highlight the benefits of diversity when constraining the overall transmit energy to $E_S = \rho T$. The fading power for the i th transmitter-receiver pair is $|h_{sd,i}|^2$, all with the same density.

When collaborative transmission is accomplished (Fig. 2(b)–(e)), the source terminal S transmits the information message for the fraction α of the overall available duration $T = 1$. During this interval, the nodes that are available for cooperation (up to N) attempt to decode the received signal. For the remaining fraction of time $1 - \alpha$, the nodes that successfully decoded the message and willing to cooperate can relay the information to the destination node D either by forwarding a copy of the message through orthogonal channels still using the same codeword (to allow for optimum combining at the receiver) or by engaging a distributed orthogonal ST coding.

In this work, we assume that each relay node might elect itself as a collaborating node if decoding is accomplished. Although straightforward, the analysis of more sophisticated relay selection strategies [11] are out of the scope of the present paper. Since performance is based on outage probability, we assume that a relay node fails in decoding during the source broadcast of duration α whenever an outage event occurs, therefore if $\alpha I_{sr} < R$ where subscript sr refers to source-to-relay links

with $I_{sr} = \log_2(1 + \mu_{sr})$ and $\mu_{sr} = \rho |h_{sr}|^2$. The probability of this event

$$\Psi(\alpha) = \Pr[\alpha I_{sr} < R] = \Pr[\mu_{sr} < 2^{R/\alpha} - 1] \quad (2)$$

depends on the time fraction α as highlighted in the argument of function $\Psi(\cdot)$ [12].

Four cooperative transmission protocols are considered as illustrated in Fig. 2(b)–(e). In order to have the paper self-contained, below we review the information theoretic outage probability for each scheme.

A. Fixed Decode and Forward Protocol—DF

Protocol description: Fixed Decode and Forward (DF) (see also [2]) is sketched in Fig. 2(b). It is based on a fixed relaying transmission where each collaborating node simply repeats the same information message. The source node broadcasts the message for the fraction $\alpha = 1/(N + 1)$, then each relay node that decoded the message (and is willing to cooperate) repeats the information symbols during the reserved¹ time fraction $1/(N + 1)$ by repeating the same codeword. The source node might have no knowledge whether the N cooperating nodes have successfully decoded the message or not, therefore relays are transparent to the source node [13] (thus simplifying the design of the medium access control MAC). Without the need for the source node to transmit directly towards the destination, this protocol is indeed a simple extension of multihop transmission (that can be, in this case, performed by multiple collaborating relays) and it requires at least one relay to fully decode the source information. At the destination, the receiver exploits the full channel state information (CSI) for each link by combining the N noisy replicas from the relays.

Outage analysis: let $k \leq N$ be the number of collaborating nodes that are willing to cooperate each for a $1/(N + 1)$ time fraction, the mutual information for the relays-to-destination link is $I_{rd}(k)/(N + 1)$, where $I_{rd}(k) = \log_2[1 + \mu_{rd}(k)]$ and $\mu_{rd}(k) = \rho \sum_{i=1}^k |h_{rd,i}|^2$ is the signal to noise ratio at the decision variable that results from the coherent combination (e.g., by maximal ratio combining) of the signal replicas coming from the k relays. Notice that each node uses the same power ρ of the source node to preserve the maximum (in case all the N nodes collaborate) total energy consumption (including the source node) $E_S = \rho T$.

Using notation I_{DF} to indicate the maximum mutual information that is achieved by DF protocol, the outage probability is [2], [3]

$$\Pr[I_{DF} < R] = \sum_{k=0}^N \mathcal{B}(k; 1/(N + 1), N) \Pr\left[\frac{I_{rd}(k)}{N + 1} < R\right] \quad (3)$$

being

$$\mathcal{B}(k; \alpha, N) = \binom{N}{k} \Psi(\alpha)^{N-k} (1 - \Psi(\alpha))^k \quad (4)$$

¹Each relay and source node is guaranteed to have an exclusive access to the wireless medium.

the probability that k nodes out of N can decode the source message within the time fraction $\alpha = 1/(N + 1)$; $\Psi(\cdot)$ is defined in (2).

B. Optimized Time Allocation Decode and Forward Protocol—O-DF

Protocol description: without requiring adaptive relaying [2], DF protocol performances can be enhanced by optimizing the time fraction α for the source node to be active so that the number of collaborating nodes is maximized. The optimized slot allocation Decode and Forward protocol (O-DF) is outlined in Fig. 2(c). The source node employs an optimized time fraction $\hat{\alpha}$ (that might be different from $1/(N + 1)$ of DF) that is needed for the broadcast phase to maximize throughput performances, or equivalently to minimize the overall outage probability. Optimization of time fraction α is a tradeoff between the maximization of the number of nodes (out of N) that successfully decode and collaborate (that would require high α) and the maximization of the time fraction $(1 - \alpha)$ reserved for the cooperative transmission. O-DF protocol requires the source node to be aware of the fading statistics for all the scheduled links by periodic control message exchange. The source node remains active for broadcast of optimized duration $\hat{\alpha}$ while the relay nodes are scheduled to repeat the message (as for DF) for a time fraction of $(1 - \hat{\alpha})/N$ each, still using the same codeword. Notice that if $\hat{\alpha} = 1/(N + 1)$ then O-DF coincides with DF.

Outage analysis: using the notation I_{ODF} to indicate the maximum mutual information that is achieved by O-DF protocol, the outage probability can be easily adapted from (3) as

$$\Pr[I_{ODF} < R] = \min_{\alpha} \sum_{k=0}^N \mathcal{B}(k; \alpha, N) \Pr\left[(1 - \alpha) \frac{I_{rd}(k)}{N} < R\right] \quad (5)$$

and it is now optimized with respect to α .

C. Selective (or Adaptive) Decode and Forward—S-DF

Protocol description: the Selective (or adaptive) Decode and Forward [2] (S-DF) is in Fig. 2(d). The source node exploits a feedback channel from the relays (by assuming a perfect feedback channel) for ACK/NACK protocol. As a basic difference to previous protocols, after a broadcast of fixed duration $\alpha = 1/(N + 1)$, transmission can reduce to a direct (from source-to-destination) link if *all* the N available nodes fail in decoding (NACK signalling). In this case, the source is allowed to use the remaining $N/(N + 1)$ time fraction by complementing the codeword with incremental redundancy. When collaborative transmission is accomplished (ACK signalling from at least one relay), each decoding relay repeats the same codeword of the source node. The destination combines all the signal replicas that can be up to $N + 1$ (included the copy from the source). In contrast to DF and ODF protocol where direct link is not used, more sophisticated MAC design is needed as the source needs to be aware of the presence of the

relays (that cannot be transparent),² moreover the destination requires CSI of the source-to-destination link.

Outage analysis: the signal-to-noise ratio at the decision variable becomes $\mu_{srd}(k) = \rho \sum_{i=1}^k |h_{rd,i}|^2 + \rho |h_{sd}|^2$ as it follows from the coherent combination of the signal replicas coming from the k relays and from the single antenna source. The mutual information for the relays-to-destination link is $I_{srd}(k)/(N+1)$, where $I_{srd}(k) = \log_2[1 + \mu_{srd}(k)]$. Using notation I_{SDF} to indicate the maximum mutual information that is achieved by S-DF protocol, the outage probability is [2]

$$\begin{aligned} \Pr[I_{SDF} < R] &= \sum_{k=1}^N \mathcal{B}\left(k; \frac{1}{N+1}, N\right) \times \Pr\left[\frac{I_{srd}(k)}{N+1} < R\right] \\ &\quad + \Psi\left(\frac{1}{N+1}\right)^N \cdot \Pr[I_{Dir}(1) < R]. \end{aligned} \quad (6)$$

Notice that if none of the relays is capable of decoding (with probability $\Psi(1/(N+1))^N$), outage probability is the same as that provided by direct transmission (1) $\Pr[I_{sd}(1) < R]$ (for $M = 1$).

D. Distributed Space-Time Coding Protocol—DST

Protocol description: Distributed Space-Time coding (DST) protocol [3] is in Fig. 2(e). In this scheme the collaborating nodes employ a distributed (orthogonal) space-time coding by simultaneously serving as a row of the ST codeword matrix [3]. After the source broadcasts for time fraction $\hat{\alpha}$ (optimized as for O-DF), the relay nodes and the source use the channel simultaneously for the whole time fraction $(1 - \hat{\alpha})$ to form a coalition where symbol level synchronization is mandatory. ST codeword matrix is therefore transmitted with each node serving as a virtual antenna.

Space-time coding techniques can be easily adapted for a distributed implementation as for space-time block codes based on orthogonal designs (orthogonal space-time coding, OSTC [10]), quasi-orthogonal [14] and space-time trellis codes [15]. A centralized (or distributed) assignment of codes to different cooperating nodes is mandatory to ensure that each collaborating node is serving as different virtual antenna.

In this paper, the OSTC is designed for the maximum of $N+1$ transmit antennas (including the source node). Even if a few of the cooperating terminals are missing due to decoding errors, the codeword matrix still remains orthogonal thus allowing the code to preserve the benefits arising from the residual diversity [16].

Outage analysis: the mutual information for the relayed link is $(1 - \hat{\alpha}) I_{srd}^{(ST)}(k)$ where, for k decoding relays, $I_{srd}^{(ST)}(k) = \log_2[1 + \mu_{srd}^{(ST)}(k)]$. The signal-to-noise ratio at the decision variable (after ST decoding) is $\mu_{srd}^{(ST)}(k) = \rho/(N+1) [\sum_{i=1}^k |h_{rd,i}|^2 + |h_{sd}|^2]$. Notice that to preserve the same total energy consumption as for DF-based protocols (and direct transmission), transmit power

of each collaborating node is now scaled to $\rho/(N+1)$. Using notation I_{DST} to indicate the maximum mutual information that is achieved by D-ST protocol, the outage probability at the destination results from the optimization over α of

$$\begin{aligned} \Pr[I_{DST} < R] &= \min_{\alpha} \sum_{k=1}^N \mathcal{B}(k; \alpha, N) \Pr\left[(1 - \alpha) I_{srd}^{(ST)}(k) < R\right] \\ &\quad + \Psi(\alpha)^N \cdot \Pr[I_{sd}(1) < R]. \end{aligned} \quad (7)$$

III. LARGE SNR OUTAGE ANALYSIS IN FADING CHANNELS

In this section, the outage analysis is extended to embrace arbitrary fading distribution over each link. Description of fading channels is in terms of inherent (fractional) diversity (d) and coding gain (c) parameters that are computed from the asymptotic behavior of the MGF of the random fading power over each link. The MGF based approach is general as it leads to a simple derivation of outage probability approximations for arbitrary number of transmitting antennas and heterogeneous/unbalanced fading over each link (technicalities are in Appendix A).

Consider a generic single-input–single-output (SISO) transmission with power ρ and an arbitrary probability density function for $|h|^2$, say $|h|^2 \sim f_{|h|^2}(w)$, that describes the severity of the fading over the considered link, the outage probability is

$$\Pr[I < R] = \int_0^{(2^R - 1)/\rho} f_{|h|^2}(w) dw. \quad (8)$$

For high ρ outage (8) is ruled by the behavior of the random fading power density around zero, $f_{|h|^2}(w)|_{w \rightarrow 0^+}$. Outage performances in fading channels are thus primarily limited by the (outage) events that cause the SNR $\mu = \rho |h|^2$ to be small with probability that depends on $f_{|h|^2}(w)|_{w \rightarrow 0^+}$. This motivates the following analysis.

The probability density function $f_{|h|^2}(w)$ can be written through the integral expansion [17]

$$f_{|h|^2}(w) = \int_0^\infty \Gamma(t+1)^{-1} \mathcal{D}^t [f_{|h|^2}(0)] w^t dt \quad (9)$$

where w^t are the basis functions and $\Gamma(x) = \int_0^\infty y^{x-1} \exp(-y) dy$ is the complete Gamma function. The t th order fractional derivative [17] $\mathcal{D}^t [f_{|h|^2}(0)]$ of $f_{|h|^2}(w)$ in $w \rightarrow 0^+$ is defined from the Laplace transform (or MGF) of $f_{|h|^2}(w)$ (i.e., $F_{|h|^2}(s) = \int_0^\infty f_{|h|^2}(w) \exp(-sw) dw$)

$$\mathcal{D}^t [f_{|h|^2}(0)] = F_0^{-1} [s^t F_{|h|^2}(s)] \quad (10)$$

where $F_0^{-1}[\cdot]$ denotes the Laplace inverse transform operator evaluated at $w \rightarrow 0^+$.

Definition 1: t^* is the order of the first nonzero fractional derivative of $f_{|h|^2}(w)$ in $w \rightarrow 0^+$ so that from (9)

$$\lim_{\varepsilon \rightarrow 0^+} \int_0^{\varepsilon^{t^* - \varepsilon}} \Gamma(t+1)^{-1} \mathcal{D}^t [f_{|h|^2}(0)] w^t dt = 0. \quad (11)$$

²Compared to DF and O-DF protocols, larger power consumption is needed as source and destination terminals are required to be active for a longer period.

The probability density function $f_{|h|^2}(w)$ can be written for w small enough as a function of t^* (Definition 1)

$$f_{|h|^2}(w) = \Gamma(t^* + 1)^{-1} \mathcal{D}^{t^*} [f_{|h|^2}(0)] w^{t^*} + o(w^{t^*}). \quad (12)$$

Substituting (12) into (8), the outage probability is a function of t^* and scales with the power ρ as

$$\Pr[I < R] \approx \frac{\mathcal{D}^{t^*} [f_{|h|^2}(0)]}{\Gamma(t^* + 2)} \left(\frac{2^R - 1}{\rho} \right)^{t^* + 1} \quad (13)$$

where we used the notation \approx to indicate that the equality holds for asymptotically large ρ .

For any given fading power distribution,³ the following proposition can be used for evaluating the order t^* of the first nonzero fractional derivative.

Proposition 1: A necessary and sufficient condition for calculating t^* is

$$\phi = \lim_{s \rightarrow \infty} s^{t^* + 1} F_{|h|^2}(s) > 0 \quad (14)$$

and finite.

Proof: Necessary condition comes from the fractional derivative definition and the initial value theorem [18]. Proving that the condition (14) is sufficient is trivial as

$$\forall \epsilon > 0 \text{ it is } \mathcal{D}^{t^* - \epsilon} [f_{|h|^2}(0)] = \lim_{s \rightarrow \infty} s^{t^* - \epsilon + 1} \cdot F_{|h|^2}(s) = 0 \quad (15)$$

therefore t^* is the smallest fractional derivative order that satisfies (14) and it is unique. \square

Remark 1: The value of t^* satisfies (14) iff

$$t^* + 1 = \lim_{s \rightarrow \infty} \left[\frac{-\log F_{|h|^2}(s)}{\log s} \right] > 0 \quad (16)$$

and finite. Proof is trivial.

The model (13) can be adapted to describe the outage probability at high SNR similarly to [1] as

$$\Pr[I < R] \approx \left(\frac{\beta N_0}{c\rho} \right)^d \quad (17)$$

where $d > 0$ is the fractional diversity $d \triangleq -\lim_{\rho \rightarrow \infty} [\log \Pr[I < R] / \log \rho]$ provided by the channel (or “inherent” diversity) and $c > 0$ is the coding gain. From (13) and (16), for a generic fading power distribution, the diversity is related to the order t^* of the first nonzero fractional derivative as

$$d \triangleq t^* + 1 = \lim_{s \rightarrow \infty} \left[\frac{-\log F_{|h|^2}(s)}{\log s} \right]. \quad (18)$$

Notice that if the MGF $F_{|h|^2}(s)$ of the channel power distribution satisfies (14) (or (16)) for some $t^* > 0$ then the provided diversity is $d > 1$ even though single antenna terminal is used for transmission.

³Fading power distribution is only required to be finite (and right-continuous) in zero: this assumption indeed holds for all the distributions that can be adjusted to fit real fading measurements.

The coding gain from (13) and (18) is

$$c \triangleq \left(\frac{\phi}{\Gamma(d+1)} \right)^{-1/d} \quad (19)$$

and ϕ is in (14). Notice that for Rayleigh fading $d = 1$ (as $t^* = 0$) and $c = E[|h|^2]$, as expected. Extended results for multi-antenna transmission that include cases with unbalanced/heterogeneous fading are given in the Appendix A. These asymptotic results for high SNR will be now adapted to the cooperative protocols.

A. Direct MISO Transmission

The outage probability at high SNR for a ST coded M antenna transmission can be written as a function of the diversity d_{sd} (18) and the coding gain c_{sd} (19) provided by the source-to-destination channel h_{sd} with $|h_{sd}|^2 \sim f_{|h_{sd}|^2}(w)$

$$\Pr[I_{sd}(M) < R] \approx \frac{\Gamma(d_{sd} + 1)^M}{\Gamma(M d_{sd} + 1)} \left(\frac{(2^R - 1) M}{c_{sd} \rho} \right)^{M d_{sd}} \quad (20)$$

(see (55) in Appendix A, with substitutions $d_i = d_{sd}$ and $c_i = c_{sd}$).

Assuming uncorrelated fading over each antenna, the (maximum) achievable diversity

$$\lim_{\rho \rightarrow \infty} \frac{-\log [\Pr(I_{sd}(M) < R)]}{\log(\rho)} = M \cdot d_{sd} \quad (21)$$

scales with the number of antennas at the source node M (as ST coding is used) and with the term d_{sd} that depends on the fading statistics.

B. Cooperative Transmission Protocols

In this section, we evaluate the terms in the outage performances at high SNR of the cooperative protocols in Section II for arbitrary fading power distributions over each link. The probability that a relay node fails in decoding $\Psi(\alpha)$ depends on the time fraction α reserved for broadcast, on the inherent diversity d_{sr} (18) and on the coding gain c_{sr} (19) provided by the source-to-relays channel h_{sr} with $|h_{sr}|^2 \sim f_{|h_{sr}|^2}(w)$. From approximation (17), the probability scales with the power ρ as

$$\Psi(\alpha) = \Pr[\alpha I_{sr} < R] \approx \left(\frac{2^{R/\alpha} - 1}{c_{sr} \rho} \right)^{d_{sr}}. \quad (22)$$

• DF protocol

Considering, for simplicity, the same fading power densities over each relay-destination pair $f_{|h_{rd,i}|^2}(w) \equiv f_{|h_{rd}|^2}(w)$, the probability $\Pr[I_{rd}(k)/(N+1) < R]$ for k cooperating nodes is similar to (20)

$$\Pr \left[\frac{I_{rd}(k)}{N+1} < R \right] = \frac{\Gamma(d_{rd} + 1)^k}{\Gamma(k d_{rd} + 1)} \left(\frac{(2^{(N+1)R} - 1)}{c_{rd} \rho} \right)^{k d_{rd}} \quad (23)$$

it depends on the diversity d_{rd} (18) and the coding gain c_{rd} (19) provided by the relay-to-destination link. The high ρ

approximation of outage probability for the fixed DF protocol is found by substituting (22) for $\alpha = 1/(N+1)$ and (23) into (3).

- *O-DF protocol* In O-DF the source node solves for the optimum time fraction based on the knowledge of the statistics of all links (namely the coding c and the diversity d pairs) that are scheduled for transmission and forwards the time allocation decisions to the N nodes. The outage probability at high SNR is found by minimizing the outage (3) over α where

$$\Pr \left[\frac{(1-\alpha)I_{rd}(k)}{N} < R \right] = \frac{\Gamma(d_{rd}+1)^k}{\Gamma(kd_{rd}+1)} \left(\frac{2^{\frac{NR}{1-\alpha}} - 1}{c_{rd}\rho} \right)^{kd_{rd}}. \quad (24)$$

For high ρ the outage probability of the O-DF protocol is found by solving the problem (5) with substitutions (22) and (24).

- *S-DF protocol* The probability $\Pr[I_{srd}(k)/(N+1) < R]$ is modelled as in (17) where diversity is provided in part by the k relays-to-destination channel d_{rd} and by the source-to-destination link d_{sd} (see Appendix A). The outage probability for S-DF protocol at high ρ is found by substituting (22) and (20) for $M = 1$ and

$$\Pr \left[\frac{I_{srd}(k)}{N+1} < R \right] \approx \left(\frac{(2^{(N+1)R} - 1)}{c_{srd}(k) \cdot \rho} \right)^{kd_{rd}+d_{sd}} \quad (25)$$

into (6), where “equivalent” coding gain for the cooperative source-relays to destination (srd) link is

$$c_{srd}(k) = \left(\frac{\Gamma(kd_{rd} + d_{sd} + 1) \cdot c_{rd}^{kd_{rd}} \cdot c_{sd}^{d_{sd}}}{\Gamma(d_{rd} + 1)^k \cdot \Gamma(d_{sd} + 1)} \right)^{1/(kd_{rd}+d_{sd})} \quad (26)$$

and depends on diversities d_{rd} , d_{sd} and coding gains c_{rd} , c_{sd} provided by all the channels that are involved in cooperative transmission.

- *D-ST protocol* The outage probability for D-ST protocol at high ρ is found by solving problem (7) with substitutions in (22) and (20) for $M = 1$ and

$$\Pr \left[(1-\alpha)I_{srd}^{(ST)}(k) < R \right] \approx \left(\frac{(2^{R/(1-\alpha)} - 1)(N+1)}{c_{srd}(k)\rho} \right)^{kd_{rd}+d_{sd}} \quad (27)$$

$c_{srd}(k)$ is in (26).

IV. ASYMPTOTIC ($\rho \rightarrow \infty$) COOPERATIVE FADING REGIONS

In this Section the concept of cooperative fading region is introduced by analyzing the outage approximation for the asymptotic case of infinite power budget, i.e., $\rho \rightarrow \infty$. Direct (noncooperative) transmission performed by a terminal equipped with M antennas is used as a reference to compare the outage performances of collaborative transmissions from N coordinated relays. Since performance is assessed for asymptotically large SNR, the regions where collaboration is beneficial are evaluated

by comparing the cooperative diversity for transmissions performed over channels with different fading distributions with the diversity provided by noncooperative transmission in Fig. 2(a). The closed form of asymptotic cooperative fading regions \mathcal{R}^∞ is thus derived regardless of the coding gains (as $\rho \rightarrow \infty$) in terms of the diversities d_{sr} , d_{rd} and d_{sd} that are computed from the respective channel power distributions, as shown in Section III. Asymptotic fading regions are also given for varying collaborating relays N and number of antenna elements M for the reference of noncooperative transmission.

A. Asymptotic Fading Regions for Fixed Relaying Schemes (DF and O-DF)

Fixed relaying based on decode and forward protocols (DF and O-DF) with $k \in [0, \dots, N]$ collaborating nodes provides the same (cooperative) diversity that follows from the analysis of outage probabilities (3) and (5) at large SNR

$$\begin{aligned} & \lim_{\rho \rightarrow \infty} \frac{-\log [\Pr(I_{DF} < R)]}{\log(\rho)} \\ &= \lim_{\rho \rightarrow \infty} \frac{-\log [\Pr(I_{ODF} < R)]}{\log(\rho)} \\ &= \min_{k \in [0, N]} \{(N-k)d_{sr} + kd_{rd}, Nd_{sr}\} \\ &= N \cdot \min \{d_{sr}, d_{rd}\}. \end{aligned} \quad (28)$$

Notice that in Rayleigh fading ($d_{sr} = d_{rd} = 1$) the diversity simply scales with the number of relaying nodes N (see [2] for the case $N = 1$).

Fading regions are evaluated in terms of benefits with respect to direct link. In other words, the cooperative diversity provided by collaborative transmissions is compared with the diversity Md_{sd} , as in (21), provided by noncooperative transmission where the source node is equipped with $M \geq 1$ antennas employing a space-time coded transmission (when $M \geq 2$). For fixed DF protocol the cooperative diversity (28) shows that for $\{d_{sr}, d_{rd}, d_{sd}, N, M\} \in \mathcal{R}_{DF}^\infty$, with

$$\mathcal{R}_{DF}^\infty = \{N \cdot \min \{d_{sr}, d_{rd}\} > M \cdot d_{sd}\} \quad (29)$$

being the asymptotic cooperative fading region, the cooperation is beneficial in providing higher diversity than the noncooperative case. From equality (28) the O-DF provides the same diversity of the DF protocol and thus

$$\mathcal{R}_{ODF}^\infty \equiv \mathcal{R}_{DF}^\infty. \quad (30)$$

Notice that, from (29), the condition $N \cdot \min \{d_{sr}, d_{rd}\} > d_{sd}$ is required to guarantee that DF and O-DF protocols can provide at least higher diversity than single antenna direct transmission ($M = 1$).

B. Asymptotic Fading Regions for Selective (or Adaptive) Relaying Schemes (S-DF and D-ST)

Considering selective relaying based decode and forward schemes (S-DF and D-ST protocols) with $(k+1) \in [1, \dots, N+1]$ collaborating nodes, the (cooperative) diversities

from the outage probabilities in (6) and (7) coincide

$$\begin{aligned} & \lim_{\rho \rightarrow \infty} \frac{-\log [\Pr(I_{\text{SDF}} < R)]}{\log(\rho)} \\ &= \lim_{\rho \rightarrow \infty} \frac{-\log [\Pr(I_{\text{DST}} < R)]}{\log(\rho)} \\ &= \min_{k \in [0, N]} \{(N-k)d_{sr} + kd_{rd}, Nd_{sr}\} + d_{sd} \\ &= d_{sd} + N \cdot \min \{d_{sr}, d_{rd}\}. \end{aligned} \quad (31)$$

In Rayleigh fading ($d_{sr} = d_{rd} = d_{sd} = 1$) diversity scales with the total number of nodes that are scheduled for transmission: $N + 1$ (see [2] for $N = 1$). The cooperative diversity (31) of the S-DF protocol, compared to the diversity provided by the direct transmission (21), shows that for $\{d_{sr}, d_{rd}, d_{sd}, N, M\} \in \mathcal{R}_{\text{SDF}}^\infty$ with

$$\mathcal{R}_{\text{SDF}}^\infty = \{N \cdot \min \{d_{sr}, d_{rd}\} > (M-1) \cdot d_{sd}\} \quad (32)$$

being the asymptotic cooperative fading region for S-DF, then cooperation is beneficial in providing higher diversity than the noncooperative case. For D-ST case, it is from (31)

$$\mathcal{R}_{\text{DST}}^\infty \equiv \mathcal{R}_{\text{SDF}}^\infty. \quad (33)$$

Notice that adaptive relaying provides a better diversity compared to single antenna direct transmission ($M = 1$) for any propagation setting as $N \cdot \min \{d_{sr}, d_{rd}\} > 0 \forall d_{sr}, d_{sd}, d_{sd}$. Adaptive relaying falls back to a single input direct transmission in the worst case that all relays are unavailable for cooperation. In all the other cases, the source node might benefit from both the direct and the relayed links.

V. COOPERATIVE FADING REGIONS FOR FINITE SNR

Outage performances of the cooperative protocols are now compared with the noncooperative case by assuming that only a finite amount of transmit energy (or SNR ρ) is available for the transmission towards the destination: for this case outage performances are ruled by diversity and coding gain pair. Cooperative fading regions at finite SNR follows by evaluating the propagation settings (diversity/coding gain pairs) that make any of the cooperative protocols (for varying number of relaying nodes N) to be beneficial in improving the outage performances with respect to the noncooperative case (where source node is equipped with M antennas).

To guarantee a fair comparison between the protocols, we set a finite transmit energy level ρT (where ρ refers equivalently to energy, or power, as $T = 1$) to satisfy a given reliability requirement for the direct (end-to-end) link. Once defined a reference energy level, the cooperative transmissions (for protocols in Fig. 2) are designed so that the sum of transmit energy consumptions for source and all relay nodes does not exceed the maximum budget ρ .

The reference energy level $\rho = \bar{\rho}$ is established from the noncooperative transmission in Fig. 2(a) once constraining a specified outage probability at the destination \bar{P}_{out} to guarantee the rate (outage capacity) \bar{R} . For a given pair $\bar{P}_{\text{out}} \ll 1$ and $\bar{R} > 1$, the required energy $\bar{\rho}$ at the M antennas source node (recall

that each antenna uses a power $\bar{\rho}/M$) can be easily derived for any fading channel by solving for ρ the equality $\Pr(I_{sd}(M) < \bar{R}) = \bar{P}_{\text{out}}$. It follows:

$$\bar{\rho} \approx \mathcal{W}(d_{sd}, M) \cdot \frac{(2^{\bar{R}} - 1) M}{c_{sd} \cdot \bar{P}_{\text{out}}^{1/M d_{sd}}} \quad (34)$$

that holds for \bar{P}_{out} small enough so that outage probability (20) holds true. Function $\mathcal{W}(d, M)$ is defined in Appendix B. Notice that the transmit power required to achieve the pair \bar{R} and \bar{P}_{out} diverges as $\bar{P}_{\text{out}} \rightarrow 0$ or $\bar{R} \rightarrow \infty$, as expected. Moreover, in case of no fading or $d_{sd} \rightarrow \infty$, since the outage probability in (20) is exactly zero for $\bar{\rho} > (2^{\bar{R}} - 1)/c_{sd}$, it is

$$\begin{aligned} \liminf_{d_{sd} \rightarrow \infty} \bar{\rho} &= \liminf_{d_{sd} \rightarrow \infty} \frac{(2^{\bar{R}} - 1) M \cdot \mathcal{W}(d_{sd}, M)}{c_{sd} \cdot \bar{P}_{\text{out}}^{1/M d_{sd}}} \\ &= (2^{\bar{R}} - 1) / c_{sd}. \end{aligned} \quad (35)$$

The same energy $\bar{\rho}$ in (34) obtained for noncooperative MISO transmission is enforced for the cooperative schemes, as $\bar{\rho}$ is the maximum energy budget that is made available for the whole transmission session. For DF and S-DF protocols this is done by allowing each relay and source node to use the channel for the same time fraction $1/(N+1)$ with the transmit power $\bar{\rho}$. Transmit energy employed by each node is thus $\bar{\rho}/(N+1)$. For O-DF and D-ST protocols, the single-antenna source node broadcasts the information symbols with energy $\hat{\alpha}\bar{\rho}$ so that up to N relay nodes decode and cooperate. Next, during the remaining time fraction $1 - \hat{\alpha}$, the residual available energy for transmission $(1 - \hat{\alpha})\bar{\rho}$ is equally split among each collaborating node by allowing each terminal either to use a transmit power $\bar{\rho}$ for a time fraction of $(1 - \hat{\alpha})/N$ (as in O-DF) or to share (now also with source, as in D-ST) the available time fraction $1 - \hat{\alpha}$ to employ ST coding with a transmit power of $\bar{\rho}/(N+1)$.

Definitions

For any cooperative transmission scheme, cooperative fading region \mathcal{R} for finite SNR is defined so that if

$$\{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M\} \in \mathcal{R}$$

then the cooperation protocol at hand has better outage performances with respect to the noncooperative case. In the following, cooperative fading regions are defined for each of the collaborative transmission protocols once the available energy at distributed multiantenna system is constrained to be the same as the direct transmission $\bar{\rho}$ in (34) with outage and rate requirements \bar{P}_{out} and \bar{R} , respectively.

Definition 2: Cooperative fading region for fixed decode and forward (DF) is

$$\mathcal{R}_{\text{DF}} \triangleq \{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M : \Pr[I_{\text{DF}} < \bar{R}] < \bar{P}_{\text{out}}\} \quad (36)$$

with $\Pr[I_{DF} < \bar{R}]$ defined in (3) and where $\rho = \bar{\rho}$. From diversity definition, it is clearly $\lim_{\bar{\rho} \rightarrow \infty} \mathcal{R}_{DF} = \mathcal{R}_{DF}^\infty$.

Definition 3: Cooperative fading region for selective decode and forward (S-DF) is

$$\mathcal{R}_{SDF} \triangleq \{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M : \Pr[I_{SDF} < \bar{R}] < \bar{P}_{out}\}, \quad (37)$$

with $\Pr[I_{SDF} < \bar{R}]$ defined in (6) and where $\rho = \bar{\rho}$. It is $\lim_{\bar{\rho} \rightarrow \infty} \mathcal{R}_{SDF} = \mathcal{R}_{SDF}^\infty$.

Definition 4: Cooperative fading region for optimized decode and forward (O-DF) is

$$\mathcal{R}_{ODF} \triangleq \{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M : \Pr[I_{ODF} < \bar{R}] < \bar{P}_{out}\} \quad (38)$$

with $\Pr[I_{ODF} < \bar{R}] = \min_\alpha \Pr[I_{DF}(\alpha) < \bar{R}]$ defined in (5) and where $\rho = \bar{\rho}$. It is $\lim_{\bar{\rho} \rightarrow \infty} \mathcal{R}_{ODF} = \mathcal{R}_{ODF}^\infty$.

Definition 5: Cooperative fading region for distributed ST coding is

$$\mathcal{R}_{DST} \triangleq \{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M : \Pr[I_{DST} < \bar{R}] < \bar{P}_{out}\} \quad (39)$$

with $\Pr[I_{DST} < \bar{R}]$ defined in (7) and where $\rho = \bar{\rho}$. It is $\lim_{\bar{\rho} \rightarrow \infty} \mathcal{R}_{DST} = \mathcal{R}_{DST}^\infty$.

A. Bounds for Cooperative Fading Regions at Finite SNR

With the aim of developing cooperative fading regions that are analytically simple to be used for practical network design, closed form bounding regions $\hat{\mathcal{R}} \supseteq \mathcal{R}$ are evaluated herein for each cooperative protocol by using the large SNR outage approximations developed in Section III. To simplify, here it is used the following comprehensive model for the bounding regions referred to the energy $\bar{\rho}$ from (34):

$$\hat{\mathcal{R}} \triangleq \{A \cdot \log_2(1/\bar{P}_{out}) + \log_2(C) > D \cdot \bar{R} + E \cdot \bar{P}_{out}^B\} \quad (40)$$

where parameters $A, B, C > 0, D > 0$ and $E > 0$ are explicitly defined in Table I for each cooperative protocol. Notice that factors A and B rule the limiting case of $\bar{\rho} \rightarrow \infty$ (that holds for asymptotic fading regions, Section IV) as $\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}} \equiv$

$\lim_{\bar{P}_{out} \rightarrow 0} \hat{\mathcal{R}} \equiv \{A > 0, B > 0\}$ (first equality comes from (34)).

Parameter C relates to the relaying methodology (that can be either fixed or selective), D and E account for the design of the time division access (that might be, in this case optimized or not) and the distributed coding technique (e.g., repetition based or orthogonal ST coding). From Table I all parameters rely on the specific outage parameters (therefore, the diversity order (d) and the coding gain (c) of each scheduled link), the number of the available (deployed) neighbor relays N and the number of antennas at the source node M when comparing to the noncooperative case.

For any given propagation scenario characterized by the (outage) parameters $(d_{sr}, c_{sr}), (d_{rd}, c_{rd})$ and (d_{sd}, c_{sd}) , since $\hat{\mathcal{R}} \supseteq \mathcal{R}$ the bounding region $\hat{\mathcal{R}}$ in (40) that is associated to a

TABLE I
COOPERATIVE FADING REGIONS PARAMETERS

Bounding regions $\{A \cdot \log_2(1/\bar{P}_{out}) + \log_2(C) > D \cdot \bar{R} + E \cdot \bar{P}_{out}^B\}$	
DF	SDF
$A_{DF} = \frac{Nd_{rd} - Md_{sd}}{M \cdot d_{sd} N d_{rd}}$	$A_{SDF} = \frac{Nd_{rd} - (M-1)d_{sd}}{M \cdot d_{sd} (Nd_{rd} + d_{sd})}$
$B_{DF} = \frac{Nd_{sr} - Md_{sd}}{M \cdot d_{sd}}$	$B_{SDF} = \frac{Nd_{sr} - (M-1)d_{sd}}{M \cdot d_{sd}}$
$C_{DF} = \frac{c_{rd} \cdot W(d_{sd}, M) \cdot M}{c_{sd} \cdot W(d_{rd}, N)}$	$C_{SDF} = c_{sr} d(N) \frac{W(d_{sd}, M) \cdot M}{c_{sd}}$
$D_{DF} = N$	$D_{SDF} = D_{DF}$
$E_{DF} = \frac{(2^{N\bar{R}} c_{sd})^{Nd_{sr}}}{Nd_{rd}} \times \left(\frac{1}{c_{sr} \cdot W(d_{sd}, M) M} \right)^{Nd_{sr}}$	$E_{SDF} = \frac{E_{DF} \cdot \left(\frac{Nd_{rd}}{Nd_{rd} + d_{sd}} \right)}{(W(d_{sd}, M) M)^{d_{sd}}}$
ODF	DST
$A_{ODF} = A_{DF}$	$A_{DST} = A_{SDF}$
$B_{ODF} = \frac{Nd_{sr} - Md_{sd}}{M d_{sd} N d_{rd}}$	$B_{DST} = \frac{Nd_{sr} - (M-1)d_{sd}}{M N d_{sr} d_{sd}}$
$C_{ODF} = C_{DF}$	$C_{DST} = \frac{C_{SDF}}{(N+1)}$
$D_{ODF} = N - 1$	$D_{DST} = 0$
$E_{ODF} = \frac{N\bar{R}^2 c_{sd} (2^{\bar{R}} - 1)^{-1}}{c_{sr} W(d_{sd}, M) M}$	$E_{DST} = \frac{E_{ODF} N^{-1}}{ W(d_{sd}, M) \cdot M ^{1/N}}$

specific cooperative transmission protocol that uses up to N relays should be considered as follows.

- i) $\{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M\} \in \hat{\mathcal{R}}$ is a *necessary* (but not sufficient⁴) condition that is required for cooperation to perform at least as if a noncooperative ST coded M antenna transmission would be employed.
- ii) $\{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M\} \notin \hat{\mathcal{R}}$ is a *sufficient* condition to prove that the cooperation is not of any benefit compared to noncooperative ST coded M antenna transmission.⁵

The following Proposition defines the closed form of the bounding regions $\hat{\mathcal{R}}_{DF}, \hat{\mathcal{R}}_{SDF}, \hat{\mathcal{R}}_{ODF}$ and $\hat{\mathcal{R}}_{DST}$ for all the considered cooperative transmission protocols that are valid for high (but still finite) $\bar{\rho}$ (or small \bar{P}_{out}) so that high SNR outage probability approximation (17) holds.

Proposition 2: For a given outage requirement \bar{P}_{out} , the following (convex) regions:

- 1) $\hat{\mathcal{R}}_{DF} = \left\{ A_{DF} \cdot \log_2(1/\bar{P}_{out}) + \log_2(C_{DF}) > D_{DF} \cdot \bar{R} + E_{DF} \cdot \bar{P}_{out}^{B_{DF}} \right\};$
- 2) $\hat{\mathcal{R}}_{SDF} = \left\{ A_{SDF} \cdot \log_2(1/\bar{P}_{out}) + \log_2(C_{SDF}) > D_{SDF} \cdot \bar{R} + E_{SDF} \cdot \bar{P}_{out}^{B_{SDF}} \right\};$
- 3) $\hat{\mathcal{R}}_{ODF} = \left\{ A_{ODF} \cdot \log_2(1/\bar{P}_{out}) + \log_2(C_{ODF}) > D_{ODF} \cdot \bar{R} + E_{ODF} \cdot \bar{P}_{out}^{B_{ODF}} \right\};$
- 4) $\hat{\mathcal{R}}_{DST} = \left\{ A_{DST} \cdot \log_2(1/\bar{P}_{out}) + \log_2(C_{DST}) > E_{DST} \cdot \bar{P}_{out}^{B_{DST}} \right\};$

are such that $\hat{\mathcal{R}}_{DF} \supset \mathcal{R}_{DF}, \hat{\mathcal{R}}_{SDF} \supset \mathcal{R}_{SDF}, \hat{\mathcal{R}}_{ODF} \supset \mathcal{R}_{ODF}$ and $\hat{\mathcal{R}}_{DST} \supset \mathcal{R}_{DST}$, parameters A, B, C, D and E are listed in Table I (notice that $D_{DST} = 0$).

Proof: The proof is in Appendix B. \square

⁴Cooperative transmissions suffer from performance degradation compared to co-located multiantenna transmissions due to imperfect synchronization/co-ordination among collaborating relays. This aspect is not considered in this paper.

⁵At least under the simplifying assumption of a ST coded transmission that uses an antenna array with uncorrelated signals.

TABLE II
DIVERSITY ORDERS (d) AND CODING GAINS (c) FOR NAKAGAMI-M AND RICE
FADING POWER DISTRIBUTIONS

Diversity (d) and coding gain (c) parameters
(Nakagami- m) $f_{ h ^2}(w; m) = \frac{w^{m-1}}{\Gamma(m-1)} \left(\frac{m}{g}\right)^m \exp\left(-\frac{mw}{g}\right)$, MGF: $F_{ h ^2}(s, m) = \left(1 + \frac{sg}{m}\right)^{-m}$
$d = m, c = \frac{g[\Gamma(m+1)]^{1/m}}{m}$
(Rice) $f_{ h ^2}(w; K) = \frac{(1+K) \exp\left(-K - \frac{(1+K)w}{g}\right)}{g} I_0\left(2\sqrt{\frac{K(1+K)w}{g}}\right)$, MGF: $F_{ h ^2}(s, K) = \frac{1+K}{1+K+sg} \exp\left(\frac{-Kgs}{1+K+sg}\right)$
$d = 1, c = \frac{g \exp(K)}{K+1}$

The following remark shows that for $\bar{\rho}$ large enough (or for \bar{P}_{out} small enough, see (34)) the bounding regions in Proposition 2 are tight in predicting the exact cooperative fading regions and approach the asymptotic fading regions.

Remark 2: For a given outage requirement \bar{P}_{out} that defines the power constraint in (34), bounding regions $\hat{\mathcal{R}}_{\text{DF}}$, $\hat{\mathcal{R}}_{\text{SDF}}$, $\hat{\mathcal{R}}_{\text{ODF}}$ and $\hat{\mathcal{R}}_{\text{DST}}$ are such that 1) $\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}}_{\text{DF}} = \mathcal{R}_{\text{DF}}^{\infty}$; 2) $\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}}_{\text{SDF}} = \mathcal{R}_{\text{SDF}}^{\infty}$; 3) $\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}}_{\text{ODF}} = \mathcal{R}_{\text{ODF}}^{\infty}$; 4) $\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}}_{\text{DST}} = \mathcal{R}_{\text{DST}}^{\infty}$.

Proof: The proof is trivial as, from Table I

$$\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}}_{\text{DF}} = \{A_{\text{DF}} > 0, B_{\text{DF}} > 0\} \equiv \mathcal{R}_{\text{DF}}^{\infty} \quad (41a)$$

$$\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}}_{\text{SDF}} = \{A_{\text{SDF}} > 0, B_{\text{SDF}} > 0\} \equiv \mathcal{R}_{\text{SDF}}^{\infty} \quad (41b)$$

$$\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}}_{\text{ODF}} = \{A_{\text{SDF}} > 0, B_{\text{SDF}} > 0\} \equiv \mathcal{R}_{\text{SDF}}^{\infty} \quad (41c)$$

$$\lim_{\bar{\rho} \rightarrow \infty} \hat{\mathcal{R}}_{\text{DST}} = \{A_{\text{DST}} > 0, B_{\text{DST}} > 0\} \equiv \mathcal{R}_{\text{DST}}^{\infty} \quad (41d)$$

VI. COOPERATIVE FADING REGIONS FOR NAKAGAMI- m FADING CHANNELS

The asymptotic and finite SNR cooperative fading regions that were developed in previous sections in terms of the inherent diversity (d) and coding gain (c) of the fading channels are now specialized for the statistical parameters of Nakagami- m fading distributions. Diversities d and coding gains c are now computed by using the mappings in Table II obtained from the asymptotic behavior of the Nakagami- m fading power MGF (for each link involved in transmission) and in term of the fading degrees m . Mappings can be similarly derived for Rice fading as in Table II and thus it follows the corresponding fading regions (not detailed in this paper, but it is just a straightforward extension). In addition, the average channel powers might be different so that $E[|h_{sd}|^2] = g_{sd}$, $E[|h_{sr}|^2] = g_{sr}$ and $E[|h_{rd}|^2] = g_{rd}$.

Experimental data show that Nakagami- m is a general parametric fading distribution $f_{|h|^2}(w; m)$ that can be adjusted to fit a variety of channel measurements [22], [23]. According to settings in Fig. 2, the fading channel between source and destination node is modeled as a Nakagami- m distribution with fading figure m_{sd} , source-to-relays is characterized by fading figure m_{sr} and collaborative transmission from relays is performed

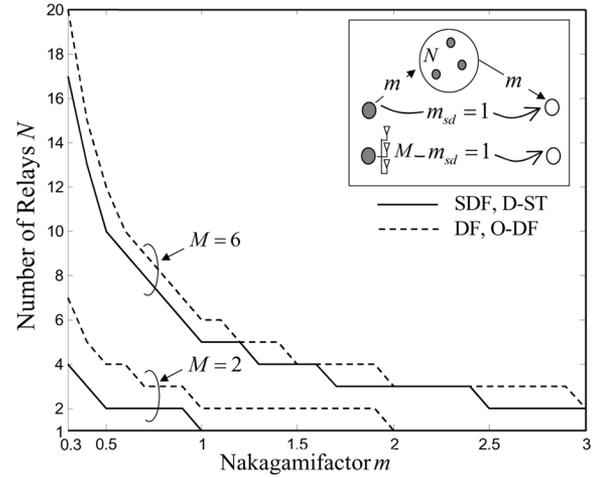


Fig. 3. Number of relays N to have same diversity performances as noncooperative direct transmission. The required N is shown versus the Nakagami factor m for the source-to-relays ($m_{sr} = m$) and relays-to-destination ($m_{rd} = m$) fading channel and for $m_{sd} = 1$, $M = 2$ and $M = 6$. Solid lines refer to adaptive relaying as for SDF and DST protocols, dashed lines analyze DF and ODF protocols.

over a channel with Nakagami factor m_{rd} . A derivation of a closed-form expression of the outage probability of distributed orthogonal space-time coding deployed over Nakagami fading channels with different channel gains and fading parameters is given in [24]. Error rate performances of DF protocols for fixed multi-antenna two-hop relay networks with Nakagami- m faded links is given in [26]. For the case of a multihop system (DF protocol with $N = 1$ relay) performance analysis is provided in [25].

A. Asymptotic Cooperative Fading Regions

Using mappings in Table II, asymptotic ($\rho \rightarrow \infty$) cooperative fading regions become:

$$\mathcal{R}_{\text{DF}}^{\infty} \equiv \mathcal{R}_{\text{ODF}}^{\infty} = \{N \cdot \min\{m_{sr}, m_{rd}\} > M \cdot m_{sd}\} \quad (42)$$

$$\mathcal{R}_{\text{SDF}}^{\infty} \equiv \mathcal{R}_{\text{DST}}^{\infty} = \{N \cdot \min\{m_{sr}, m_{rd}\} > (M-1) \cdot m_{sd}\}. \quad (43)$$

As a numerical example, in Fig. 3 we compute the required number of relays N to have same (cooperative) diversity performances as if the source node would employ ST-coded transmission with M antennas. The required N is computed for fixed and adaptive relaying to satisfy conditions (42) and (43) with strict equality. Results are shown versus the (same) Nakagami factor m for the source-to-relays (here $m_{sr} = m$) and relays-to-destination ($m_{rd} = m$) fading channels and for $m_{sd} = 1$ (Rayleigh fading), $M = 2$ and $M = 6$. Solid lines refer to adaptive relaying as for SDF and DST protocols, dashed lines analyze DF and ODF protocols. The most promising propagation settings that can motivate the use of cooperative transmissions (e.g., that require a lower number of relays compared to the number of antennas for noncooperative transmission) primarily arise when the Nakagami factor m for the relayed links is larger than 1 (so that milder diffusive fading is experienced compared to direct source-to-destination link).

TABLE III
PARAMETERS FOR COOPERATIVE FADING REGIONS IN
NAKAGAMI- m ENVIRONMENTS

Bounding regions for Nakagami- m fading	
DF	SDF
$A_{DF} = \frac{Nm_{rd} - Mm_{sd}}{M \cdot m_{sd} N m_{rd}}$	$A_{SDF} = \frac{Nm_{rd} - (M-1)m_{sd}}{M \cdot m_{sd} (Nm_{rd} + m_{sd})}$
$B_{DF} = \frac{Nm_{sr} - Mm_{sd}}{M \cdot m_{sd}}$	$B_{SDF} = \frac{Nm_{sr} - (M-1)m_{sd}}{M \cdot m_{sd}}$
$C_{DF} = N \frac{g_{rd}}{m_{rd} g_{sd}}$	$C_{SDF} = C_{DF} \frac{m_{rd}}{g_{rd}} \times$ $\times \left(\frac{g_{rd}}{m_{rd}} \right)^{(Nm_{rd} + m_{sd})} \times$ $\times \left(\frac{g_{sd}}{m_{sd}} \right)^{(Nm_{rd} + m_{sd})}$
$D_{DF} = N$	$D_{SDF} = D_{DF}$
$E_{DF} = \frac{1}{Nm_{rd}} \left(\frac{2^{N\bar{R}} g_{sd}}{g_{sr}} \right)^{Nm_{sr}}$	$E_{SDF} = \left(\frac{\Gamma(Mm_{sd} + 1)^{1/M}}{\Gamma(m_{sd} + 1) M^{m_{sd}}} \right) \times$ $\times \left(\frac{Nm_{rd} \cdot E_{DF}}{Nm_{rd} + m_{sd}} \right)$
ODF	DST
$A_{ODF} = A_{DF}$	$A_{DST} = A_{SDF}$
$B_{ODF} = \frac{Nm_{sr} - Mm_{sd}}{MN \cdot m_{sd} m_{sr}}$	$B_{DST} = \frac{Nm_{sr} - (M-1)m_{sd}}{MN \cdot m_{sd} m_{sr}}$
$C_{ODF} = C_{DF}$	$C_{DST} = \frac{C_{SDF}}{(N+1)}$
$D_{ODF} = N - 1$	$D_{DST} = 0$
$E_{ODF} = \frac{NRm_{sr}g_{sd}}{g_{sr}m_{sr} \cdot (2^{\bar{R}} - 1)}$	$E_{DST} = \frac{E_{ODF}}{N}$

B. Cooperative Fading Regions at Finite SNR

When considering a finite SNR, by using substitutions in Table II, the required transmit power for an outage reliability of \bar{P}_{out} and $m_{sd} < \infty$ follows from (34) as

$$\bar{\rho} \approx \frac{(2^{\bar{R}} - 1) M m_{sd}}{g_{sd} [\Gamma(M m_{sd} + 1) \bar{P}_{out}]^{1/M m_{sd}}}. \quad (44)$$

The corresponding cooperative fading regions for the considered protocols are defined as

$$\mathcal{R}_{DF} = \left\{ m_{sr}, m_{rd}, m_{sd}, N, M : \Pr[I_{DF} < \bar{R}] < \bar{P}_{out} \right\} \quad (45a)$$

$$\mathcal{R}_{SDF} = \left\{ m_{sr}, m_{rd}, m_{sd}, N, M : \Pr[I_{SDF} < \bar{R}] < \bar{P}_{out} \right\} \quad (45b)$$

$$\mathcal{R}_{ODF} = \left\{ m_{sr}, m_{rd}, m_{sd}, N, M : \Pr[I_{ODF} < \bar{R}] < \bar{P}_{out} \right\} \quad (45c)$$

$$\mathcal{R}_{DST} = \left\{ m_{sr}, m_{rd}, m_{sd}, N, M : \Pr[I_{DST} < \bar{R}] < \bar{P}_{out} \right\} \quad (45d)$$

while the corresponding bounding regions $\hat{\mathcal{R}}$ are found by using the model (40) and the parameters listed in Table III

C. Numerical Analysis

In this section, cooperative fading regions are evaluated numerically for Nakagami- m fading by assuming that a finite power budget (or finite SNR as in Section V) is available for transmission (cooperative fading regions for Rice fading could be similarly derived from Table II).

The available power $\bar{\rho}$ from (34) is computed from the following source-to-destination link requirements: $\bar{P}_{out} = 10^{-6}$

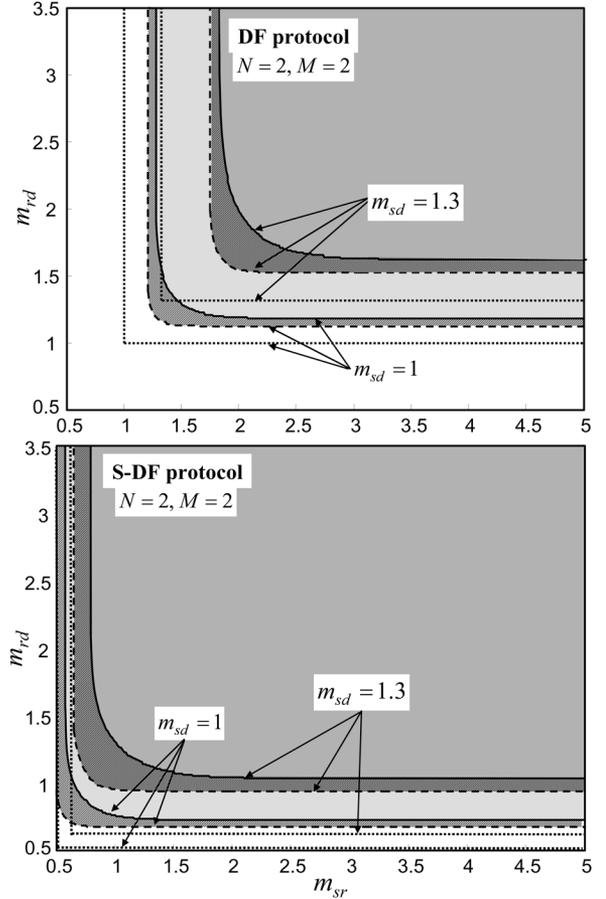


Fig. 4. Finite SNR ($\bar{P}_{out} = 10^{-6}$, $\bar{R} = 2$ [b/s/Hz], $\ell_{sr} = \ell_{rd} = \ell_{sd}/2$) cooperative fading regions \mathcal{R}_{DF} (on top) and \mathcal{R}_{SDF} (at bottom) for fixed DF and S-DF protocols (boundaries are in solid lines) with varying Nakagami factors m_{sr} , m_{rd} , for $m_{sd} = 1$ (Rayleigh fading) and $m_{sd} = 1.3$. $N = 2$ relays are available for cooperation. For direct transmission (Fig. 2(a)), used as reference, the source node is equipped with $M = N$ antennas. Bounding region $\hat{\mathcal{R}}_{DF}$ (with $\hat{\mathcal{R}}_{DF} \supseteq \mathcal{R}_{DF}$) and $\hat{\mathcal{R}}_{SDF}$ (with $\hat{\mathcal{R}}_{SDF} \supseteq \mathcal{R}_{SDF}$) limits are in dashed lines, dotted lines refer to the asymptotic regions $\mathcal{R}_{DF}^{\infty}$ and $\mathcal{R}_{SDF}^{\infty}$ for $\bar{\rho} \rightarrow \infty$.

and $\bar{R} = 2$ b/s/Hz.⁶ Path loss model is with exponent $\kappa = 2$. Distance between the source and the destination is set to $\ell_{sd} = 1$ (Fig. 2(a)) so that $g_{sd} = \ell_{sd}^{-\kappa} = 1$. Source-to-relays and relays-to-the-destination distances are $\ell_{sr} = \ell = 1/2$ and $\ell_{rd} = 1 - \ell$ (Fig. 2), respectively. Average channel powers for each cooperative link are balanced: $g_{sr} = g_{rd} = (1/2)^{-\kappa}$.

The cooperative fading regions are analyzed (Figs. 4, 5) by comparing the performances of the collaborative transmission protocols considered in this paper with respect to direct link with M antennas (Fig. 2(a)) for varying fading parameters and number of relay terminals N . Cooperative regions are illustrated as shaded areas in all figures to highlight the propagation settings for source-to-relays and relays-to-destination links where cooperation is beneficial in providing enhanced performances with respect to the multi-antenna case in Fig. 2(a) used as reference.

Fig. 4 shows finite SNR cooperative fading regions (in terms of boundaries delimited by solid lines) \mathcal{R}_{DF} (top) and \mathcal{R}_{SDF}

⁶Lower outage probability \bar{P}_{out} is for higher power (e.g., $\bar{\rho} \geq 15$ dB for $\bar{P}_{out} = 10^{-6}$) and thus the tightness of the bounding regions to the exact regions.

(bottom) for fixed DF and S-DF protocols, respectively. Nakagami factors (m_{sr} , m_{rd}) are ranging from $m = 0.5$, (one sided Gaussian distribution) to ($m_{sr} = 5$, $m_{rd} = 3.5$) for two values of source-to-destination fading figure $m_{sd} = 1$ and $m_{sd} = 1.3$. $N = 2$ relays are available for cooperation (recall that for S-DF the source joins the collaborative transmission), performance comparison is given with respect to a noncooperative system (Fig. 2(a)) where the source node is equipped with $M = 2$ antennas (thus employing Alamouti ST coding). Dashed curves limit the approximated fading regions $\hat{\mathcal{R}}_{\text{DF}}$ (on the left) and $\hat{\mathcal{R}}_{\text{SDF}}$ (on the right) that can be evaluated in closed form. Dotted lines refer to the boundaries of the asymptotic regions for $\bar{\rho} \rightarrow \infty$.

Adaptive relaying strategy supported by the S-DF protocol is shown to either enhance the diversity and provide larger fading regions in all cases. As an example, let us consider from Fig. 4 a channel between source and relays that exhibits NLOS propagation ($m_{sr} = 1$). Cooperative transmission when employing DF protocol, is unfeasible for any m_{rd} and m_{sd} , while for S-DF protocol the collaborative transmission is preferable to multi-antenna whenever $m_{rd} \geq 0.7$ for $m_{sd} = 1$, and $m_{rd} \geq 1.3$ for $m_{sd} = 1.3$.

In Fig. 5 the boundaries of the cooperative fading (bounding) regions for finite SNR are shown in dashed lines for the O-DF $\hat{\mathcal{R}}_{\text{ODF}}$ (top) and D-ST $\hat{\mathcal{R}}_{\text{DST}}$ (bottom). Markers indicate the (exact) fading region boundaries that are obtained by optimizing the achievable outage over α for O-DF (as in Definition 3) and for D-ST (as in Definition 4). $N = 2$ relays are used for collaboration, performance comparison is given with respect to multi-antenna case with $M = 2$ (to ease of comparison with Fig. 4), but also for $M = 3$, $M = 4$ and Rayleigh fading ($m_{sd} = 1$). O-DF enhances the performances of DF as it provides wider regions. However, compared to the case $M = 2$ in Fig. 4, S-DF is still preferable as it exploits the full diversity in the number of transmitting nodes. D-ST protocol allows for distributed coding among the collaborating terminals and it provides the largest regions in all cases by trading with the complexity of a required symbol-level synchronization among cooperating nodes.

VII. ON DIVERSITY-MULTIPLEXING TRADEOFF IN FADING CHANNELS

In previous Sections we evaluated the cooperative diversity performances and (asymptotic) cooperative fading regions for arbitrary fading distributions and fixed rate R . This approach is typically used to analyze the performances of communication systems that need to guarantee a fixed end-to-end throughput and to maximize the reliability of the data stream. Instead of simply increasing the rate, another way to evaluate the diversity performances for high spectral efficiency is to allow the rate to increase with the SNR [3]:

$$R = R(\rho) = r \times \log(1 + \rho) \quad (46)$$

where r is the spectral efficiency of the protocol normalized to the maximum achievable spectral efficiency [2] (or multiplexing gain [9]). Parametrizing protocol performances with respect to r reveals a tradeoff between the provided diversity gain and the

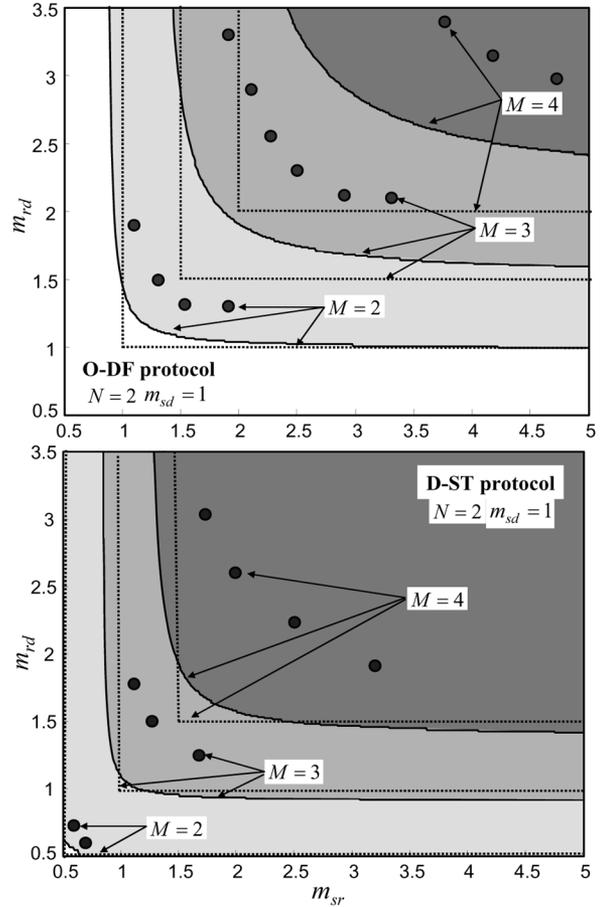


Fig. 5. Bounds (dashed lines) of the cooperative fading region for finite SNR ($\bar{P}_{\text{out}} = 10^{-6}$, $\bar{R} = 2$ [b/s/Hz], $\ell_{sr} = \ell_{rd} = \ell_{sd}/2$) for the O-DF, $\hat{\mathcal{R}}_{\text{ODF}}$ (on top), and D-ST, $\hat{\mathcal{R}}_{\text{DST}}$ (at bottom), with $N = 2$ relays. Markers indicate the (exact) fading region boundaries from Definition 3 (for O-DF, $\mathcal{R}_{\text{ODF}} \subseteq \hat{\mathcal{R}}_{\text{ODF}}$) and Definition 4 (for D-ST, $\mathcal{R}_{\text{DST}} \subseteq \hat{\mathcal{R}}_{\text{DST}}$). Dotted lines are boundaries of the asymptotic regions $\hat{\mathcal{R}}_{\text{ODF}}^{\infty}$ and $\hat{\mathcal{R}}_{\text{DST}}^{\infty}$ for $\bar{\rho} \rightarrow \infty$. For direct transmission (Fig. 2(a)) the source is equipped with $M = 2$ (for comparison with Fig. 4), three and four antennas, $m_{sd} = 1$.

achievable normalized spectral efficiency, thus serving as an effective method to assess asymptotic performances of cooperative protocols at high spectral efficiency [2], compared to noncooperative MISO performance used as benchmark.

Definition 6: A cooperative transmission protocol (e.g., DF, O-DF, S-DF, and DST) can achieve a normalized spectral efficiency (or spatial multiplexing gain) r and diversity gain $d(r)$ if the transmission rate $R(\rho)$ (46) and the outage probability $\Pr[I < R(\rho)]$ satisfy the following equalities:

$$r = \lim_{\rho \rightarrow \infty} \frac{R(\rho)}{\log(\rho)} \quad (47)$$

$$d(r) = \lim_{\rho \rightarrow \infty} \frac{-\log[\Pr(I < r \times \log(1 + \rho))]}{\log(\rho)}, \quad (48)$$

for mutual information I referred to any of the cooperative protocols in Section II. For each value of r , $d(r)$ refers to the achievable diversity that is offered by the particular transmission methodology. The spatial redundancy (or diversity $d(r)$) that can be provided to the data stream can be traded for higher spectral efficiency (or higher multiplexing gain r).

TABLE IV
DIVERSITY-MULTIPLEXING TRADEOFFS FOR COOPERATIVE
TRANSMISSION PROTOCOLS

DF protocol
$d(r) = \lim_{\rho \rightarrow \infty} \frac{-\log[\Pr(I_{DF} < R(\rho))] =}{\log(\rho)} = N \min\{d_{sr}, d_{rd}\} \times [1 - (N+1)r]$
ODF protocol
$d(r) = \lim_{\rho \rightarrow \infty} \frac{-\log[\Pr(I_{ODF} < R(\rho))] =}{\log(\rho)} = N \min\{d_{sr}(1 - \frac{r}{\hat{\alpha}}), d_{rd}(1 - N\frac{r}{1-\hat{\alpha}})\}$
SDF protocol
$d(r) = \lim_{\rho \rightarrow \infty} \frac{-\log[\Pr(I_{SDF} < R(\rho))] =}{\log(\rho)} = [d_{sd} + N \min\{d_{sr}, d_{rd}\}] \times [1 - (N+1)r]$
DST protocol
$d(r) = \lim_{\rho \rightarrow \infty} \frac{-\log[\Pr(I_{DST} < R(\rho))] =}{\log(\rho)} = \min\{d_{sd}(1-r) + Nd_{sr} \times [1 - \frac{r}{\hat{\alpha}}], (Nd_{rd} + d_{sd}) [1 - \frac{r}{1-\hat{\alpha}}]\}$

For direct (noncooperative) MISO ST coded transmission diversity-multiplexing tradeoff (DMT) is evaluated in terms of the achievable diversity gain versus the normalized spectral efficiency [20]:

$$d(r) = \lim_{\rho \rightarrow \infty} \frac{-\log[\Pr(I_{sd}(M) < r \times \log(\rho))]}{\log(\rho)} = Md_{sd}(1-r) \quad (49)$$

with $r < 1$. Although the main focus here is the analysis of the cooperative diversity in arbitrary fading channels, in this section we shed a light on how to extend the DMT [19] to arbitrary distributed fading impairments. To simplify the reasoning, we consider the same simple scenario (as for Sections IV and V) where the receiving node has single antenna [21]. Using (46), together with large SNR outage approximations in Section III and (cooperative) diversities in Section IV, DMTs are derived by computing the available (cooperative) diversity $d(r)$ for each protocol as a function of r as in [19] and [3]. For each case, values are in Table IV, derivations are based on asymptotic outage analysis in Section IV.

Using the same propagation setting as in Fig. 3 for Nakagami- m fading, Fig. 6 shows the function $d(r)$ from Table IV for DF (dashed lines), DST (solid lines) and direct MISO ST-coded transmission (49) (dotted lines) for varying Nakagami factors m , $M = 3$ and $N = 2$. Moderate diffusive component of fading (that is modeled by $m > 1$) provides larger diversity that can be traded for higher normalized spectral efficiency. DST protocol offers larger diversity (and most favorable diversity-multiplexing tradeoff) compared to fixed relaying schemes and thus it can be used for higher spectral efficiencies as shown in [3]. Collaborative transmission can provide better tradeoff compared to direct (dotted lines) when the relayed path exhibit a lower diffusive fading compared to Rayleigh ($m > 1$). This is necessary to counterbalance the larger overhead the distributed processing must account for compared to noncooperative transmission. More specifically, according to the propagation setting under study and using the tradeoffs in Table IV, DF protocol can provide more favorable DMT compared to direct transmission (see markers in Fig. 6)

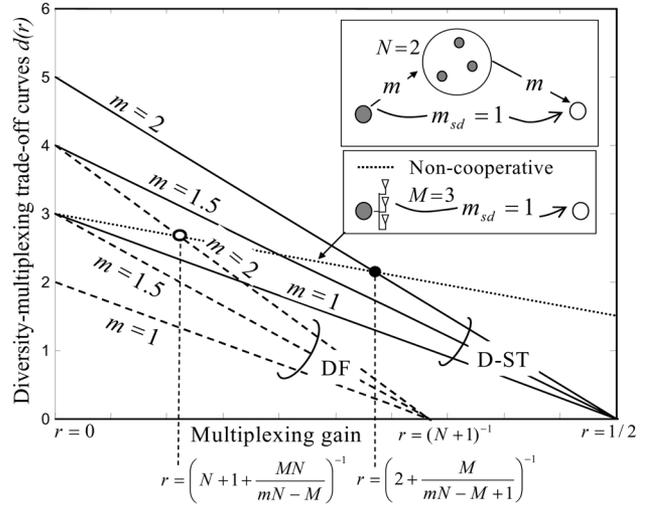


Fig. 6. Diversity-multiplexing tradeoff curves (see Table IV) for noncooperative ST coded transmission with $M = 3$ antennas (dotted lines), DF (dashed lines) and DST ($\hat{\alpha} = 1/2$ [3] in solid lines) protocols with $N = 2$ relays. Results are shown for varying Nakagami factors for relayed links m .

when $r < [N + 1 + NM/(Nm - M)]^{-1}$, condition for DST protocol prescribes $r < [2 + M/(Nm + 1 - M)]^{-1}$.

A. Asymptotic Cooperative Fading Regions and DMT

The asymptotic cooperative fading region concept introduced in Section IV is revisited here using DMT framework. Achievable (cooperative) diversity gain for cooperative protocols $d(r)$ in Table IV is compared to that provided by the direct transmission (49). This equivalence shows that for $\{d_{sr}, d_{rd}, d_{sd}, N, M\} \in \mathcal{R}^\infty(r)$ with

$$\mathcal{R}^\infty(r) = \{d(r) > Md_{sd}(1-r)\} \quad (50)$$

then collaborative transmission is beneficial in providing higher diversity gain than the noncooperative case for the same value of normalized spectral efficiency r . For each protocol, asymptotic cooperative fading region $\mathcal{R}^\infty(r)$ can thus be defined in general for any target efficiency r .

Remark 3: Given the spectral efficiency requirements $r = r_1$ and $r = r_2$, asymptotic cooperative fading regions for each collaborative transmission protocol are such that

$$\mathcal{R}^\infty(r_1) \supseteq \mathcal{R}^\infty(r_2) \forall r_1 \leq r_2 < 1. \quad (51)$$

Asymptotic regions clearly converge to those derived in Section IV for fixed rate, as expected.

VIII. MAIN CONCLUSIONS AND REMARKS

In this paper we considered transmission protocols based on the cooperation of a number of decode and forward relaying nodes. Since short range communications often experience propagation environments that cannot be simply modeled by Rayleigh fading, benefits of cooperative transmission in terms of provided diversity and outage probability are analyzed for arbitrary fading distributions by introducing the concept of cooperative fading regions. These regions define the statistical propagation settings that make collaboration among terminals

beneficial in improving performances compared to multi-antenna noncooperative MISO transmission (used as reference setting). We proposed a general framework for parametrizing the outage probability for any fading distribution in decode and forward protocols (extension to amplify and forward is in [8]). Closed form bounding fading regions that can be used for practical network design are based on large SNR approximation of outage derived from the asymptotic behavior of the MGF of the fading power distribution. This analysis is intended to get more insight into how the propagation settings can influence the performances of collaborative transmission.

Although in Rayleigh fading the performances of decode and forward protocols are severely affected by the inter-user (or source-to-relay) channel reliability [2], if less severe fading for the same link can be assumed (e.g., allowing for practical scenarios where the source node is mostly in short range with the relays) or if relays are fixed and in strategic locations, regions can be derived where cooperation is beneficial in providing comparable performances to noncooperative MISO schemes. The main results drawn from cooperative fading regions analysis in Nakagami- m and Rice fading environments (although specific numerical examples for the Rice fading case are not shown) can be summarized as follows.

- i) Collaborative transmission (that use either fixed or adaptive relaying) exhibits the highest improvements, compared to multi-antenna transmission, whenever the source-to-destination link suffers from highly diffusive fading (NLOS propagation, or $m_{sd} \leq 1$).
- ii) Multihop transmission (with $N = 1$ for DF and O-DF protocols, see Fig. 2(b) and (c)) has comparable performances as those from Alamouti ST coding ($M = 2$) in Rayleigh fading only when *both* the source-to-relays *and* relays-to-destination links exhibit mild fading with significant LOS component (e.g., for LOS factors above 9 dB), or large Nakagami factor (see also [25]).
- iii) Collaborative transmission ($N > 1$) that is employed by repetition based schemes (DF, O-DF and S-DF) is beneficial with respect to direct transmission if *either* the source-to-relay (as in ad-hoc networking) *or* the relays-to-destination (for fixed relays applications) channels are impaired by a fading that is less severe than Rayleigh (e.g., with a meaningful LOS component, or Nakagami factor $m > 1$).⁷
- iv) D-ST protocol exploits more powerful coding so that requirements on fading statistics to enhance performances of direct transmission become less stringent for *all* settings.

APPENDIX

A. Outage Probability Approximation for Distributed MISO Links

Here we consider the outage performances in fading channels for a transmission that is accomplished by a distributed

⁷In ad-hoc applications the relays are typically in short range with the source node and thus benefit from milder fading compared to Rayleigh. Instead, in fixed relay applications the relays-to-destination channels might exhibit a fading with marginal diffusive component (due to careful selection of relay sites).

antenna system. Let M transmitting nodes (or N for relays) using the same power level of ρ each, the instantaneous SNR over each transmitter-receiver pair is $\mu_i = \rho |h_i|^2$, where $|h_i|^2 \sim f_{|h_i|^2}(w)$ and $f_{|h_i|^2}(w)$ is an arbitrary link-dependent distribution. After ST decoding (in case of multi-antenna transmission or D-ST protocol) or coherent combining of multiple signal replicas (in case of repetition based operation: DF, S-DF and O-DF) the SNR at the decision variable has the form $\mu(M) = \sum_{i=1}^M \mu_i = \rho \sum_{i=1}^M |h_i|^2$ with $\sum_{i=1}^M |h_i|^2 \sim g_M(w) = f_{|h_1|^2}(w) * \dots * f_{|h_M|^2}(w)$ (* stands for convolution).

The outage probability is $\Pr[\mu(M) < \beta] = \Pr\left[\sum_{i=1}^M |h_i|^2 < \beta/\rho\right]$, where β is defined according to the specific transmission scheme (see Section II). As for (13) and large enough ρ , it is

$$\Pr[\mu(M) < \beta] \approx \frac{\mathcal{D}^{\tau^*} [f_{g_M}(0)]}{\Gamma(\tau^* + 2)} \left(\frac{\beta}{\rho}\right)^{\tau^* + 1} \quad (52)$$

and τ^* is the smallest order for which $\mathcal{D}^{\tau^*} [f_{g_M}(0)] > 0$.

The following corollary to Proposition 1 defines τ^* as a function of the smallest fractional derivative orders t_i^* for which $\mathcal{D}^{t_i^*} [f_{|h_i|^2}(0)] > 0$ and finite.

Corollary 1: Defining t_i^* for $i = 1, \dots, M$ as the smallest fractional derivative orders (see Definition 1) satisfying (14) with $h = h_i$ (assume $f_{|h_i|^2}(w) \in \mathcal{C}^\infty$ at least for $w \rightarrow 0^+$), then, in (52), it is

$$\tau^* = \sum_{i=1}^M t_i^* + M - 1. \quad (53)$$

Proof: The MGF of density $g_M(w)$ is $G_M(s)$ and it reduces, for uncorrelated channels over each transmitting and receiving pairs, to the product of the MGFs of each $|h_i|^2$ random variable, thus $G_M(s) = \prod_{i=1}^M F_{|h_i|^2}(s)$. Being $\mathcal{D}^{\tau^*} [g_M(0)] = \lim_{s \rightarrow \infty} s^{\tau^* + 1} G_M(s)$, for τ^* in (53) it is

$$\lim_{s \rightarrow \infty} s^{\tau^* + 1} G_M(s) = \prod_{i=1}^M \mathcal{D}^{t_i^*} [f_{|h_i|^2}(0)] > 0 \quad (54)$$

and it is thus proved from Proposition 1. \square

The diversities $d_i = t_i^* + 1$ in (18) and the coding gains c_i (19) provided by the channel between the i th antenna and the destination defines the outage probability for the $M \times 1$ link that scales as (for ρ large enough)

$$\Pr[\mu(M) < \beta] \approx \left(\frac{\prod_{i=1}^M \Gamma(d_i + 1)}{\Gamma(\sum_{i=1}^M d_i + 1) \cdot \prod_{i=1}^M c_i^{d_i}} \right) \left(\frac{\beta}{\rho}\right)^{\sum_{i=1}^M d_i}. \quad (55)$$

If (55) is rewritten to fit the model (17), using Corollary 3 the fractional diversity is

$$d \triangleq \lim_{\rho \rightarrow \infty} \frac{\log \Pr[\mu(M) < \beta]}{\log \rho} = \sum_{i=1}^M d_i = M + \sum_{i=1}^M t_i^* \quad (56)$$

notice that we might interpret (56) as the combination of the diversity term M that is provided by the multiantenna transmission, and the contribution $\sum_{i=1}^M t_i^*$ that is related to the channel statistics over each link. The coding gain is

$$c \triangleq \left(\frac{\Gamma \left(\sum_{i=1}^M d_i + 1 \right) \cdot \prod_{i=1}^M c_i^{d_i}}{\prod_{i=1}^M \Gamma(d_i + 1)} \right)^{1/\sum_{i=1}^M d_i}. \quad (57)$$

Results (56), (57) and (55) generalize the derivations in [1] and [7] and represent the background for evaluating outage performances in cooperative transmissions with arbitrarily distributed fading channels. Notice that extension to distributed MIMO transmissions [30] is straightforward.

B. Proof of Proposition 2

- i) DF: From results in Section III by letting $\mathcal{B}(N; \alpha, N) \approx 1$ it is

$$\begin{aligned} \Pr(I_{\text{DF}} < \bar{R}) & \\ & \geq \frac{\Gamma(d_{rd} + 1)^N}{\Gamma(Nd_{rd} + 1)} \left(\frac{2^{(N+1)\bar{R}} - 1}{c_{rd} \cdot \bar{\rho}} \right)^{Nd_{rd}} + \Psi \left(\frac{1}{N+1} \right)^N \\ & = \mathcal{P}_{\text{out}}, \end{aligned} \quad (58)$$

equality holds asymptotically for high $\bar{\rho}$. By substituting (34) into (58) the cooperative fading bounding region is now $\hat{\mathcal{R}}_{\text{DF}} = \{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M : \mathcal{P}_{\text{out}} < \bar{\mathcal{P}}_{\text{out}}\}$ and after straightforward computations it can be reduced to the following:

$$\left\{ A_{\text{DF}} \cdot \log_2(1/\bar{\mathcal{P}}_{\text{out}}) + \log_2(C_{\text{DF}}) > D_{\text{DF}} \bar{R} \right. \\ \left. - \frac{1}{Nd_{rd}} \log_2 \left[1 - Nd_{rd} E_{\text{DF}} \cdot \bar{\mathcal{P}}_{\text{out}} \right], \right. \\ \left. Nd_{rd} E_{\text{DF}} \cdot \bar{\mathcal{P}}_{\text{out}}^{B_{\text{DF}}} < 1 \right\}. \quad (59)$$

Right side of inequality (59) can be finally expanded (assuming finite values for the diversity and coding gain pairs for each channel) to have a region that fits with the model (40).

- ii) S-DF: From results in Section III for $\mathcal{B}(N; \alpha, N) \approx 1$ it is

$$\begin{aligned} \Pr(I_{\text{SDF}} < \bar{R}) & > \left(\frac{2^{(N+1)\bar{R}} - 1}{c_{srd}(N) \cdot \bar{\rho}} \right)^{Nd_{rd} + d_{sd}} \\ & + \Psi \left(\frac{1}{N+1} \right)^N \cdot \left(\frac{2^{\bar{R}} - 1}{c_{sd} \bar{\rho}} \right)^{d_{sd}} \\ & = \mathcal{P}_{\text{out}}. \end{aligned} \quad (60)$$

By substituting (34) into (60) cooperative fading bounding region is now $\hat{\mathcal{R}}_{\text{SDF}} = \{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M : \mathcal{P}_{\text{out}} < \bar{\mathcal{P}}_{\text{out}}\}$

and it can be reduced to the following:

$$\left\{ A_{\text{SDF}} \cdot \log_2(1/\bar{\mathcal{P}}_{\text{out}}) + \log_2(C_{\text{SDF}}) > D_{\text{SDF}} \bar{R} \right. \\ \left. - \frac{1}{Nd_{rd} + d_{sd}} \log_2 \left[1 - (Nd_{rd} + d_{sd}) E_{\text{SDF}} \cdot \bar{\mathcal{P}}_{\text{out}}^{B_{\text{SDF}}} \right], \right. \\ \left. (Nd_{rd} + d_{sd}) E_{\text{SDF}} \cdot \bar{\mathcal{P}}_{\text{out}}^{B_{\text{SDF}}} < 1 \right\}. \quad (61)$$

Right side of inequality (61) can be finally expanded (assuming finite diversity and coding gain pairs for each channel) to have a region that fits with the model (40). The following result will be used for deriving the bounding regions for O-DF and D-ST protocols. The minimum required time fraction α for successful decoding of one relay with a given probability $p_1 \ll 1$ is found by solving $\Psi(\alpha) = p_1$ from (22) for α

$$\begin{aligned} \alpha(p_1) & = \Psi^{-1}(p_1) \\ & = \min \left\{ 1, \frac{R}{\log_2 \left(1 + \frac{c_{sr} \mathcal{W}(d_{sd}, M) \cdot M \cdot (2^R - 1) P_{\text{out}}^{-1/Md_{sd}}}{c_{sd} p_1^{-1/d_{sr}}} \right)} \right\} \end{aligned} \quad (62)$$

where

$$\mathcal{W}(d, M) = \Gamma(d+1)^{1/d} \cdot \Gamma(Md+1)^{-1/Md}. \quad (63)$$

- iii) O-DF: The outage probability at the destination is found by optimizing the time fraction α as in (5), or equivalently by using (62) to minimize the achievable outage over p_1

$$\begin{aligned} \Pr(I_{\text{ODF}} < \bar{R}) & \\ & = \min_{p_1 < \bar{\mathcal{P}}_{\text{out}}^{1/N}} \sum_{k=1}^N \mathcal{B}(k; \alpha(p_1), N) \\ & \quad \times \frac{\Gamma(d_{rd} + 1)^k}{\Gamma(kd_{rd} + 1)} \left(\frac{2^{\frac{N\bar{R}}{1-\alpha(p_1)}} - 1}{c_{rd} \cdot \bar{\rho}} \right)^{kd_{rd}} + p_1^N \end{aligned} \quad (64)$$

where $\bar{\rho}$ is from (34) and we used outage approximation results in Section III. Feasible set for p_1 is found by constraining $p_1^N < \bar{\mathcal{P}}_{\text{out}}$: the probability that all relay nodes fail in decoding must be at least less or equal to the reliability requirement $\bar{\mathcal{P}}_{\text{out}}$. By letting $\mathcal{B}(N; \alpha, N) \approx 1$ it is now

$$\begin{aligned} \Pr(I_{\text{ODF}} < \bar{R}) & \\ & \geq \min_{p_1 < \bar{\mathcal{P}}_{\text{out}}^{1/N}} \frac{\Gamma(d_{rd} + 1)^N}{\Gamma(Nd_{rd} + 1)} \left(\frac{2^{\frac{N\bar{R}}{1-\alpha(p_1)}} - 1}{c_{rd} \cdot \bar{\rho}} \right)^{Nd_{rd}} \\ & + p_1^N > \mathcal{P}_{\text{out}} \end{aligned} \quad (65)$$

and

$$\mathcal{P}_{\text{out}} = \frac{\Gamma(d_{rd} + 1)^N}{\Gamma(Nd_{rd} + 1)} \left[\left(2^{\frac{NR}{1-\alpha(\bar{P}_{\text{out}}^{1/N})}} - 1 \right) \frac{1}{c_{rd} \cdot \bar{\rho}} \right]^{Nd_{rd}}. \quad (66)$$

Cooperative fading bounding region is now $\hat{\mathcal{R}}_{\text{ODF}} = \{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M : \mathcal{P}_{\text{out}} < \bar{P}_{\text{out}}\}$ and it can be reduced to the following

$$\left\{ A_{\text{ODF}} \cdot \log_2(1/\bar{P}_{\text{out}}) + \log_2(C_{\text{ODF}}) \right. \\ \left. > \frac{N-1 + \alpha(\bar{P}_{\text{out}}^{1/N})}{1 - \alpha(\bar{P}_{\text{out}}^{1/N})} \bar{R} \right\}. \quad (67)$$

Having, for small \bar{P}_{out} , and $B_{\text{ODF}} > 0$

$$\alpha(\bar{P}_{\text{out}}^{1/N}) \sim \alpha^*(\bar{P}_{\text{out}}^{1/N}) = \frac{\bar{R}c_{sd} \cdot \bar{P}_{\text{out}}^{B_{\text{ODF}}}}{c_{sr} \mathcal{W}(d_{sd}, M) \cdot M \cdot (2^{\bar{R}} - 1)} \quad (68)$$

(in case $B_{\text{ODF}} < 0$ it is $\alpha(\bar{P}_{\text{out}}^{1/N}) < \alpha^*(\bar{P}_{\text{out}}^{1/N})$ and $\alpha^*(\bar{P}_{\text{out}}^{1/N}) \leq \alpha(\bar{P}_{\text{out}}^{1/N})(1 + \alpha^*(\bar{P}_{\text{out}}^{1/N}))$) then, by noticing that $1/[1 - \alpha(\bar{P}_{\text{out}}^{1/N})] \geq 1 + \alpha^*(\bar{P}_{\text{out}}^{1/N})$, region (67) can be easily bounded to fit the model (40) after straightforward algebraic computation.

iv) D-ST: The outage probability at the destination is similarly found by using (62) to minimize the achievable outage over p_1 ,

$$\Pr(I_{\text{DST}} < \bar{R}) = \min_{p_1 < \bar{P}_{\text{out}}^{1/N}} \sum_{k=1}^N \mathcal{B}(k; \alpha(p_1), N) \\ \times \left(\frac{2^{\frac{\bar{R}}{1-\alpha(p_1)}} - 1}{\frac{c_{sr}d(N) \cdot \bar{\rho}}{N+1}} \right)^{k \cdot d_{rd} + d_{sd}} + p_1^N \left(\frac{2^{\bar{R}} - 1}{c_{sd} \bar{\rho}} \right)^{d_{sd}}$$

where we used outage approximation results in Section III with $\bar{\rho}$ from (34) and $\bar{P}_{\text{out}} = \bar{P}_{\text{out}}^{(M-1)/M} [\mathcal{W}(d_{sd}, M) \cdot M]$ so that $p_1^N \left(\frac{2^{\bar{R}} - 1}{c_{sd} \bar{\rho}} \right)^{d_{sd}} < \bar{P}_{\text{out}}$. By letting $\mathcal{B}(N; \alpha, N) \approx 1$ it is now

$$\Pr(I_{\text{DST}} < R) > \min_{p_1 < \bar{P}_{\text{out}}^{1/N}} \left(\frac{2^{\frac{\bar{R}}{1-\alpha(p_1)}} - 1}{\frac{c_{sr}d(N) \cdot \bar{\rho}}{N+1}} \right)^{N \cdot d_{rd} + d_{sd}} \\ + p_1^N \left(\frac{2^{\bar{R}} - 1}{c_{sd} \bar{\rho}} \right)^{d_{sd}} > \mathcal{P}_{\text{out}} \quad (69)$$

and

$$\mathcal{P}_{\text{out}} = \left((N+1) \frac{2^{\bar{R}/[1-\alpha(\bar{P}_{\text{out}}^{1/N})]} - 1}{c_{sr}d(N) \cdot \bar{\rho}} \right)^{N \cdot d_{rd} + d_{sd}}. \quad (70)$$

Cooperative fading bounding region is now $\hat{\mathcal{R}}_{\text{DST}} = \{(d_{sr}, c_{sr}), (d_{rd}, c_{rd}), (d_{sd}, c_{sd}), N, M : \mathcal{P}_{\text{out}} < \bar{P}_{\text{out}}\}$ and it can be reduced to the following:

$$\left\{ A_{\text{DST}} \cdot \log_2(1/\bar{P}_{\text{out}}) + \log_2(C_{\text{DST}}) \right. \\ \left. > \alpha(\bar{P}_{\text{out}}^{1/N}) \cdot \bar{R} / \left[1 - \alpha(\bar{P}_{\text{out}}^{1/N}) \right] \right\}. \quad (71)$$

Having, for small \bar{P}_{out} , and $B_{\text{DST}} > 0$

$$\alpha(\bar{P}_{\text{out}}^{1/N}) \sim \alpha^*(\bar{P}_{\text{out}}^{1/N}) \\ = \frac{\bar{R}c_{sd} \cdot \bar{P}_{\text{out}}^{B_{\text{DST}}}}{c_{sr} [\mathcal{W}(d_{sd}, M) \cdot M]^{\frac{N+1}{N}} \cdot (2^{\bar{R}} - 1)} \quad (72)$$

then, region (71) can be bounded (following the same technicalities as for ODF) to fit the model (40).

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