

Distributed Orthogonal Space-Time Coding: Design and Outage Analysis for Randomized Cooperation

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Abstract—In this paper we consider a cooperative wireless network where each terminal communicates to a destination node with the aid of multiple relaying nodes. The focus is on cooperative transmission protocols that are based on the simultaneous transmission by a number of cooperating nodes. Outage performances are analyzed by assuming a Distributed Randomized Orthogonal Space-Time Coding scheme (DR-OSTC) to be employed by the relaying terminals during the transmission session. The DR-OSTC scheme requires that each cooperating node chooses *randomly and independently* to serve as one of the space-time virtual antennas. By avoiding any pre-defined terminal-to-codeword mapping, the random selection of the space-time codewords substantially reduces the needed control overhead with respect to other distributed space-time coding strategies simplifying the node coordination task. According to this scheme, in this paper it is tackled the problem of designing both the minimum number of cooperating nodes M and the spatial dimension L of the space-time code matrix so as to meet a specific outage probability requirement at the destination. Outage performances are also analyzed by developing simple but effective design rules tailored for two cooperative transmission protocols in realistic propagation environments.

Index Terms—Distributed space-time coding, cooperative diversity, space-time coding design, ad-hoc and sensors networks, wireless networks.

I. INTRODUCTION

IN wireless networks the channel fading is one of the main source of impairment that could be mitigated through the use of appropriate spatial redundancy, also known as diversity. When terminals are equipped with multiple antenna array at the transmitters, space-time coding schemes can be designed to achieve a maximum diversity order at minimum possible loss in rate. When the use of nodes with multiple antennas is not a viable solution due to hardware, size and cost constraints, diversity can still be achieved by exploiting the cooperation among the antennas of different terminals so as to benefit from cooperative diversity. In [1] protocols are proposed that involve each cooperating terminal to repeat (by decoding and forwarding or by simply amplifying the received symbols) the user message using orthogonal subchannels (i.e., by employing time (TDMA) division, code (CDMA) division or frequency (FDMA) division multiple access). Since cooperation occurs through a bandwidth inefficient repetition based

scheme, recently authors in [2] have investigated an alternative cooperation methodology referred to as *coded cooperation*. This scheme is based on incremental redundancy where the codeword to be transmitted by the relays is partitioned among each cooperating node so as to achieve significant performance gains (in terms of bit error rate (BER) and spectral efficiency) and a great degree of flexibility by allowing different code rates and partitions among nodes.

Within the class of relay based distributed coding techniques, the conventional distributed version of the orthogonal space-time coding scheme (herein referred to as D-OSTC) requires a set up phase where each cooperating node has to be informed on which space-time codeword should be employed. Although a loss in the overall rate has to be accounted for, D-OSTC is appropriate even when the number of transmitters is unknown, as in case of cooperative networks (see [3], [4] and [5]). However, to ensure that every codeword is correctly assigned to a different active cooperating node, the coordinated D-OSTC protocol exhibits a low throughput as it requires the exact knowledge of which node is available to collaborate and therefore it involves control overhead (e.g., acknowledgement messages exchange among the collaborating nodes) that increases with the required diversity degree and node mobility. Moreover, since each cooperating terminal is distributed in space, performance degradation up to $2 \div 3$ dB in terms of signal to noise ratio (SNR) can be observed (both in terms of bit error rate (BER) [5] and outage probability) with respect to the OSTC bound for a conventional multiple antenna transmitter [6]. This difference is namely due to unequal average channel powers of the links between each cooperating terminal and the destination node. Recently it has been shown that performances of D-OSTC schemes can be significantly enhanced by allowing opportunistic selection of the most favorable subset of terminals among the cooperative set [7].

This paper builds upon the framework introduced in [8] to analyze the outage performances for the Distributed Randomized Orthogonal Space-Time Coding scheme (DR-OSTC) to be employed in cooperative wireless networks. The idea of designing and optimizing a distributed space-time coding strategy based on the simultaneous transmission of a linear combination of the space-time coding matrix was independently proposed by [8] and [9]. In particular, within this class of D-OSTC, randomized OSTC was introduced in [8] and [10] to eliminate the problem of centralized assignment of codes to different cooperating nodes still preserving the spectral efficiency and the diversity of an orthogonal space-time code. In this paper, we focus on a simple randomized transmission

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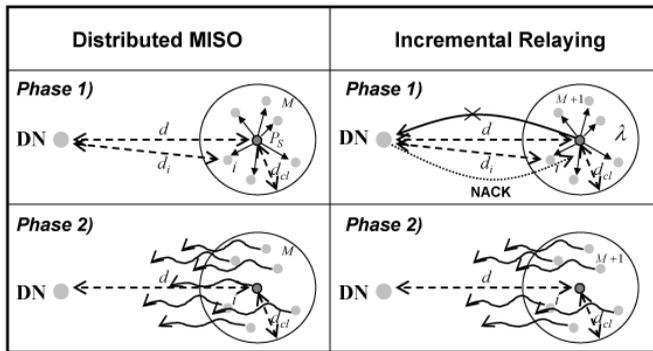


Fig. 1. Distributed MISO (on the left) and Incremental relaying (on the right) protocols and simulation environment.

scheme (referred to as antenna selection in [8]) where, for a given space-time codeword \mathbf{C} with dimensions $L \times p$, each cooperating node chooses *randomly and independently* to use one of the rows of \mathbf{C} and thus to serve as one of the L space-time virtual antennas. In [10] more complex soft-randomization rules (that require each node to transmit a random linear combination of the rows of \mathbf{C}) are proposed to achieve the full code diversity even with a moderate number of relays. Since DR-OSTC strategy does not require the exact knowledge of which specific node is going to collaborate (at least at the transmitting side), it significantly reduces the required amount of control overhead when compared to the D-OSTC scheme. In other words, DR-OSTC does not require any pre-defined “terminal-to-code matrix row mapping” and thus avoids any acknowledgement message exchange among the collaborating nodes. However, the randomized approach involves specific design strategies for both the complex orthogonal space-time codewords \mathbf{C} and the number of cooperating relay nodes M so as to achieve the required performances in terms of outage probability.

The original contribution of this paper is twofold: first a novel analytic model to evaluate the outage probability for the DR-OSTC scheme is developed. Next, the problem of designing both the minimum number of cooperating nodes M and the spatial dimension L of the space-time code matrix \mathbf{C} is tackled so as to meet a specific outage probability requirement at the destination node. The performance results of the randomized scheme are compared with those obtained from coordinated distributed space-time coding schemes (D-OSTC) and conventional multiple antenna based OSTC.

A. Cooperative Protocols

Here it is briefly reviewed two cooperative wireless network protocols that will be considered in the following analysis (see figure 1). In the first one, (left side of figure 1) referred to as *distributed MISO* (Multiple Input Single Output) [11] [12], we assume a transmitter cluster to be composed of a number of sensor nodes communicating with a receiving destination node (DN). The protocol consists of two phases. First, the data to be transmitted is broadcast by the source node, so that $M - 1$ active nodes within the cluster can decode the data to be relayed during the MISO transmission (in general, the

set of $M - 1$ active nodes is a subset of the total number of nodes in the cluster). In the second phase, the data is transmitted through the $M \times 1$ MISO channel (including the source node) employing a distributed-randomized orthogonal space-time coding (DR-OSTC) scheme. Due to half duplex constraint, both phases have to be carried out in two orthogonal channels (e.g., by time division) resulting in a reduced spectral efficiency with respect to a multiple antenna based MISO system.

The second investigated protocol, also referred to as *incremental relaying* (or collaborative retransmission) [1] [13], is designed so that whenever a source starts a transmission session to the DN, all the terminals covered by the same DN potentially receive the transmission intended for the destination. The DN indicates success or failure of transmission by broadcasting a single bit of feedback (ACK/NACK feedback). If a transmission failure occurs (NACK), M terminals act as relay nodes by decoding and forwarding what received from the source thus employing a distributed orthogonal space-time coding scheme. Loosely speaking, the incremental relaying protocol may be viewed as a virtual hybrid automatic-repeat-request (H-ARQ) policy where the retransmissions are not generally originated from the source, but they could come from any relay that overhears and decodes earlier transmitted blocks. This provides a spatial diversity effect that might be beneficial in block fading environments where time-diversity could not be exploited [13]. By allowing cooperation to be exploited only during a source-to-DN link failure, in [13] the protocol is shown to provide a better tradeoff between throughput and energy consumption, even if performances can be severely influenced by an imperfect feedback channel.

The paper is organized as follows. After system model definitions (Sect. II), the outage analysis of OSTC and of cooperative based schemes (D-OSTC) is reviewed in Sect. III. In Sect. IV the outage probability is derived by assuming a DR-OSTC protocol to be employed at the cooperating terminals, results are then corroborated by simulations. Code design (namely in terms of degree of cooperation) with outage probability constraints is dealt with in Sect. V. Finally, Sect. VI sheds a light on how the proposed analysis can help in designing a realistic planning for both the proposed cooperative protocols. As notation, in what follows subscripts R and D are used so as to refer to DR-OSTC and D-OSTC schemes, respectively.

II. SYSTEM MODEL

In both the link models in fig. 1, it is assumed that the worst fading conditions are experienced by the links towards the DN node while the links between the source node and the in-range relays exhibit a strong line of sight component. Under these settings, the disk model [14] applies so that relays can fully decode the transmissions for cooperation if the received signal to noise ratio (SNR) exceeds a minimum threshold (further details will be given in Sect. VI). Each channel gain between the m -th cooperating terminal (for $m = 1, \dots, M$) and the destination node is modelled by $h_m \sim CN(0, \Gamma_m)$ (Rayleigh fading model with $E[h_m h_n^*] = 0$, for $\forall m \neq n$) and it is known by the destination node (DN), but it is not available to the M transmitting nodes. According to the outlined cooperative

protocols, we assume that M nodes covered by the same DN node (or belonging to the same Basic Service Set (BSS)) correctly receive and decode the source message intended for the DN node and thus belong to the decoding (or cooperative) set.

Let ρ be the SNR at the decision variable of the DN, the outage probability relative to β is

$$\Pr(\rho < \beta) = F(\beta) = \mathcal{P}_{out}, \quad (1)$$

and it depends on the Cumulative Density Function (CDF) of SNR $F(\beta)$ for a specified minimum SNR β that guarantees an acceptable performance. Models for the maximum rate R per unit bandwidth that can be supported over a link with minimum SNR β can be written as [15]: $\beta = (2^R - 1) / K$, where K (with $0 < K < 1$) relies on the selected coding-modulation scheme and the required BER. From an information theoretic perspective, the link towards the DN node is in outage for SNR ρ if the channel capacity $C(\rho) = \log_2(1 + \rho) < R$, then $\beta = 2^R - 1$. Since the link reliability between the source and the destination is guaranteed as long as $\rho \geq \beta$, the average transmission rate when considering many bursts is $(1 - \mathcal{P}_{out})R$.

In the following P_r denotes the overall power budget available at the distributed multi-antenna system composed by all the M cooperating nodes. For practical convenience, it is expected that each cooperating node allocates the same amount of power during the cooperative transmission (in case of a distributed MISO protocol) or retransmission (in case of an incremental relaying protocol) session. However, in this paper the transmit power level at each cooperating node is scaled with respect to M as P_r/M in order to highlight the diversity gain rather than the gain for increasing the overall received power (when M nodes are cooperating) and thus guaranteeing a fair comparison between the two space-time coding schemes. During each cooperative transmission (or retransmission), we assume perfect synchronization among the M transmitting nodes. It is understood that a negligible frequency offset and a perfect timing advance is no longer guaranteed in realistic conditions so that performance degradation is experienced [6].

Under static fading assumption over the whole codeword duration p , the received signal at the DN node \mathbf{y} ($p \times 1$) is

$$\mathbf{y}^T = \sqrt{\frac{P_r}{M}} \mathbf{v}^T \mathbf{C} + \mathbf{n}^T. \quad (2)$$

For both schemes the dimension $L \times p$ of the code matrix \mathbf{C} can be decided according to the required outage probability requirements \mathcal{P}_{out} (and β). Here, the spatial (L) and temporal (p) dimensions of \mathbf{C} are denoted by the pair (L_D, p_D) and (L_R, p_R) for D-OSTC and DR-OSTC, respectively. Since each row of \mathbf{C} is a linear combination of the unit power q source symbols $\{s_1, \dots, s_q\}$ ($E[|s_i|^2] = 1$) ($q = q_D$ for the D-OSTC and $q = q_R$ for the DR-OSTC) and of their complex conjugates such that $\mathbf{C}\mathbf{C}^H = \kappa \mathbf{I}_{L \times L}$ (and κ is a constant), the space-time code rate is $R_L = q/p \leq 1$. According to the selected inner code and modulation format, with the symbol rate R_s , the overall transmission rate per unit bandwidth is $R = R_s R_L$. In (2) $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{p \times p})$ is the unit power additive white Gaussian noise vector with dimensions $p \times 1$.

According to the specific node-to-code mapping, the vector $\mathbf{v} = [v_1, \dots, v_L]^T$ is specifically defined for the D-OSTC scheme (in Sect. III) and for the DR-OSTC (in Sect. IV), it contains a linear combination of the channel gains $\mathbf{h} = [h_1, \dots, h_M]^T \sim \mathcal{CN}(\mathbf{0}, \text{diag}(\boldsymbol{\Gamma}_M))$ of the links between each cooperating terminal and the DN. Average channel powers of each link towards the destination node are collected into the M length vector $\boldsymbol{\Gamma}_M = [\Gamma_1, \dots, \Gamma_M]^T$ that models arbitrarily unbalanced links as we assume $\Gamma_i \neq \Gamma_j$. Due to path loss, shadowing and node distance towards the DN node, each set of fading powers $\{\Gamma_i\}_{i=1}^M$ is a realization of i.i.d. random variable with probability density function $p_\Gamma(\Gamma)$.

To ease the comparison, we refer the performance with respect to the average SNR at the receiving DN

$$\bar{\rho} = E_\Gamma[\Gamma]P_r, \quad (3)$$

as it would represent the SNR experienced by the DN if the cooperative transmission (or retransmission) was performed by only one single relay that use the whole power P_r with average attenuation $E_\Gamma[\Gamma] = \int x p_\Gamma(x) dx$.

III. REVIEW OF D-OSTC OUTAGE PERFORMANCES

In order to make the paper self-contained, here we briefly review the outage performances of Orthogonal Space-Time Coding schemes that take advantage of a distributed virtual antenna system (for a detailed discussion, the reader might refer to [3], [16] and [17]). A Distributed-OSTC scheme requires each cooperating terminal to be assigned (e.g., by the source node) to a specific row of the space-time codeword matrix \mathbf{C} . Herein the following assumptions hold: *i*) to minimize the control overhead, the number of cooperating nodes M_D is equal to the selected spatial dimension of the code L_D , so that $M_D = L_D$; *ii*) a centralized mapping is decided so that a different space-time codeword is assigned to each cooperating node, therefore vector \mathbf{v} in (2) coincides with the channel gain vector \mathbf{h} : $\mathbf{v} = \mathbf{h}$.

By using the channel decoupling property and by allowing a loss in the overall code rate, it can be shown that decoding of distributed OSTC schemes with maximum likelihood (ML) detection can be still decomposed into q_D scalar detection problems for the unknown symbols s_1, \dots, s_{q_D} (see [4] and [18]). For each symbol, the instantaneous SNR at the decision variable is (recalling that $E[|s_i|^2] = 1$):

$$\rho = \frac{P_r}{M_D} \|\mathbf{h}\|^2. \quad (4)$$

The probability density function (pdf) of the SNR ρ at the DN node differs from the chi-squared distribution as for conventional OSTC [4] in that the average channel powers $\{\Gamma_i\}_{i=1}^{L_D}$ of the links are unbalanced. From equation (14.5.26) in [19] and recalling that $M_D = L_D$, the pdf of ρ conditioned on $\boldsymbol{\Gamma}_{L_D} = [\Gamma_1, \dots, \Gamma_{L_D}]^T$ can be written:

$$f^D(\rho | \boldsymbol{\Gamma}_{L_D}) = \sum_{i=1}^{L_D} \frac{A_i(\boldsymbol{\Gamma}_{L_D}) L_D}{\Gamma_i P_r} \exp\left(-\frac{\rho L_D}{\Gamma_i P_r}\right) \quad (5)$$

where $A_i(\Gamma_{L_D}) = \prod_{\ell \neq i}^L \frac{\Gamma_i}{\Gamma_i - \Gamma_\ell}$. The outage probability conditioned on Γ_{L_D} reads:

$$P_{out}^D(L|\Gamma_{L_D}) = \int_0^\beta f^D(\rho|\Gamma_{L_D})d\rho = \sum_{i=1}^{L_D} A_i(\Gamma_{L_D}) \left(1 - \exp\left(-\frac{\beta L_D}{\Gamma_i P_r}\right) \right). \quad (6)$$

By averaging with respect to the pdf of Γ_{L_D} , the unconditional outage probability over the distributions of unbalanced relay-to-destination channel powers becomes:

$$P_{out}^D(L_D) = E_{\Gamma}[P_{out}^D(\Gamma_{L_D})] = \int p_{\Gamma}(\Gamma_{L_D}) P_{out}^D(L_D|\Gamma_{L_D}) d\Gamma_{L_D} \quad (7)$$

where $p_{\Gamma}(\Gamma_{L_D}) = \prod_{i=1}^{L_D} p_{\Gamma}(\Gamma_i)$ is the joint distribution of the average channel powers for each link.

Under large SNR $\bar{\rho} = E_{\Gamma}[\Gamma]P_r \gg L_D\beta$, the outage probability for D-OSTC (7) simplifies to (Appendix VIII-A):

$$P_{out}^D(L_D) \simeq \frac{1}{L_D!} \left(\frac{L_D\beta}{\bar{\rho}\epsilon} \right)^{L_D}. \quad (8)$$

Outage in (8) is similar to the outage performances of multiple antenna OSTC scheme except for SNR degradation $\epsilon \in (0, 1]$ that accounts for unequal average fading powers on each link. More specifically, the degradation factor ϵ for i.i.d. power is (Appendix VIII-A):

$$\epsilon = \frac{1}{E_{\Gamma}[\Gamma] \cdot E_{\Gamma}[\frac{1}{\Gamma}]}. \quad (9)$$

However, following the definition of diversity order in [20], the D-OSTC protocol still achieves the full diversity in the number of virtual antennas L_D as from (8) it is:

$$\lim_{\bar{\rho} \rightarrow \infty} \frac{-\log(P_{out}^D(L_D))}{\log(\bar{\rho})} = L_D. \quad (10)$$

IV. DISTRIBUTED RANDOMIZED-OSTC (DR-OSTC) - ANTENNA SELECTION

Differently from D-OSTC, the DR-OSTC consists in that each of the M_R collaborating nodes randomly and independently chooses to serve as one of the L_R virtual antennas by selecting a codeword from a space-time matrix \mathbf{C} now with dimensions $L_R \times p_R$. Performance depends on the probability that more terminals select the same space-time codeword thus yielding a loss of diversity. To focus on this, let k_i be the (random) number of terminals that are using the i -th row out of the L_R rows of the coding matrix \mathbf{C} such that $\sum_{i=1}^{L_R} k_i = M_R$, the probability of a specific terminal-to-code matrix row assignment $\mathbf{k} = [k_1, \dots, k_{L_R}]$ can be modelled by a multinomial distribution:

$$\Pr(\mathbf{k}) = \Pr(k_1, \dots, k_{L_R}) = \frac{M_R!}{\prod_{i=1}^{L_R} k_i!} \left(\frac{1}{L_R} \right)^{M_R}. \quad (11)$$

For any assignment vector \mathbf{k} , we use $\mathbf{G}(\mathbf{k}) \in \mathcal{G}_{\mathbf{k}}$ as an $L_R \times M_R$ selection matrix associated to a specific random terminal-to-code matrix row mapping. The (ℓ, m) entry of $\mathbf{G}(\mathbf{k})$ is:

$$g_{\ell, m} = \begin{cases} 1 & \text{if the } m\text{-th terminal chooses to serve} \\ & \text{as the } \ell\text{-th coding matrix row} \\ 0 & \text{otherwise} \end{cases}. \quad (12)$$

Every row and column of $\mathbf{G}(\mathbf{k})$ is constrained so that

$$\mathbf{G}(\mathbf{k})\mathbf{G}(\mathbf{k})^T = \text{diag}(k_1, \dots, k_{L_R}), \quad (13)$$

or equivalently $\sum_{\ell=1}^{L_R} g_{\ell, m} = 1, \forall m = 1, \dots, M_R$, and $\sum_{m=1}^{M_R} g_{\ell, m} = k_{\ell}, \forall \ell = 1, \dots, L_R$.

The number of cooperating terminals is greater than the spatial code dimension L_R , so that $M_R \geq L_R$. This setting guarantees for DR-OSTC protocol that the probability $\Pr(\text{rank}[\mathbf{G}(\mathbf{k})\mathbf{G}(\mathbf{k})^T] = L_R)$ that all the L_R space-time codewords are chosen at least by one terminal is greater than zero. From (13) the vector $\mathbf{v} = [v_1, \dots, v_{L_R}]^T$ in (2) can be written as a linear combination of *independent* entries of \mathbf{h} from selection matrix:

$$\mathbf{v} = \mathbf{G}(\mathbf{k}) \cdot \mathbf{h} \sim CN(\mathbf{0}, \text{diag}(\tilde{\Gamma}_{L_R})), \quad (14)$$

here $\tilde{\Gamma}_{L_R} = \mathbf{G}(\mathbf{k}) \cdot \Gamma_{M_R}$ is the L_R -length vector whose i -th element contains the sum of all the k_i channel powers of those terminals that choose to use the same i -th coding matrix row:

$$\tilde{\Gamma}_i = [\tilde{\Gamma}_{L_R}]_i = \sum_{m=1}^{M_R} g_{i, m} \Gamma_m, \text{ for } i = 1, \dots, L_R.$$

For each symbol, the instantaneous SNR ρ at the decision variable is thus:

$$\rho = \frac{P_r}{M_R} \|\mathbf{v}\|^2. \quad (15)$$

The outage probability conditioned on $\tilde{\Gamma}_{L_R}$ is:

$$P_{out}^R(M_R, L_R|\tilde{\Gamma}_{L_R}) = \sum_{i=1}^{L_R} A_i(\tilde{\Gamma}_{L_R}) \left(1 - \exp\left(-\frac{\beta M_R}{\tilde{\Gamma}_i P_r}\right) \right), \quad (16)$$

where $A_i(\tilde{\Gamma}_{L_R}) = \prod_{\ell \neq i}^{L_R} \frac{\tilde{\Gamma}_i}{\tilde{\Gamma}_i - \tilde{\Gamma}_\ell}$. Since: *i*) the random selection $\mathbf{G}(\mathbf{k})$ is independent on power Γ_{M_R} ; *ii*) the probability density function of $\tilde{\Gamma}_{L_R} = \mathbf{G}(\mathbf{k}) \cdot \Gamma_{M_R}$ is $p(\tilde{\Gamma}_{L_R})$, and it can be decoupled as $\Pr(\mathbf{k}) \cdot \Pr(\mathbf{G}(\mathbf{k})|\mathbf{k}) \cdot p_{\Gamma}(\Gamma_{M_R})$; *iii*) for a given \mathbf{k} value the integral $\int p_{\Gamma}(\Gamma_{M_R}) P_{out}^R(M_R, L_R|\mathbf{G}(\mathbf{k}) \cdot \Gamma_{M_R}) d\Gamma_{M_R}$ does not depend on the particular choice of $\mathbf{G}(\mathbf{k}) \in \mathcal{G}_{\mathbf{k}}$, then it follows that the outage probability averaged over the set $\tilde{\Gamma}_{L_R}$ becomes decoupled into:

$$P_{out}^R(M_R, L_R) = \sum_{\mathbf{k}, k_1 + \dots + k_{L_R} = M_R} \Pr(\mathbf{k}) \cdot \int p_{\Gamma}(\Gamma_{M_R}) P_{out}^R(M_R, L_R|\mathbf{G}(\mathbf{k}) \cdot \Gamma_{M_R}) d\Gamma_{M_R}. \quad (17)$$

The summation in (17) includes all assignments $k_i \geq 0$ such that $\sum_{i=1}^{L_R} k_i = M_R$ as well as the channel power distribution $p_{\Gamma}(\Gamma_{M_R}) = \prod_{m=1}^{M_R} p_{\Gamma}(\Gamma_m)$.

In [8], it is shown that the DR-OSTC protocol achieves the maximum diversity order of L_R when the average signal to noise ratio $\bar{\rho}$ is lower than a threshold value $\bar{\rho}_t$ and $M_R \rightarrow \infty$. In Appendix VIII-B, it is evaluated analytically the threshold

$$\bar{\rho}_t = \beta L_R \left(\frac{M_R - 1}{L_R - 1} \right), \quad (18)$$

when $\bar{\rho} \gg L_R\beta$ and $M_R > L_R$ in terms of the selected code matrix spatial dimension L_R and of the number of cooperating terminals M_R . For any finite value of M_R , and $\bar{\rho} \rightarrow \infty$ the diversity reduces to 1 as the least likely event that all the users

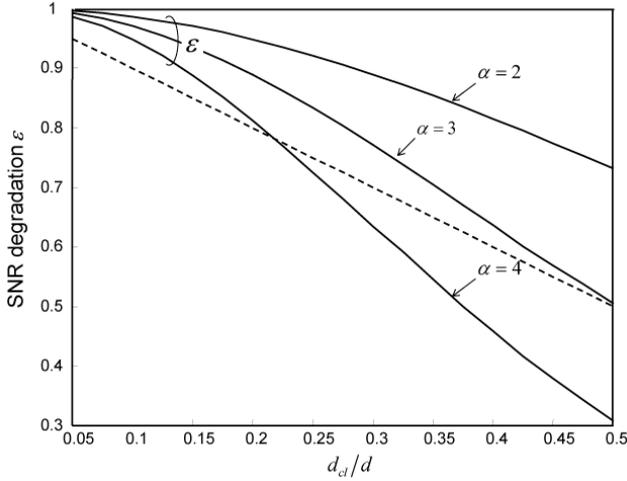


Fig. 2. SNR degradation ϵ versus the ratio d_{cl}/d for path loss exponent $\alpha = 2, 3, 4$. Dashed line refer to the approximation $1 - d_{cl}/d$.

select the same code (i.e., $k_i = M_R$ and $k_j = 0$ for $\forall j \neq i$) has a dominating effect on the outage performances. At finite SNR¹ $\bar{\rho}$ as far as $\bar{\rho} < \bar{\rho}_t$ (but still $\bar{\rho} \gg L_R \beta$ for the limit (18) to hold) the diversity order [21] of the DR-OSTC scheme can be upperbounded as (Appendix VIII-B)

$$\frac{-\log(P_{out}^R(M_R, L_R))}{\log(\bar{\rho})} < L_R 2^{-(1/M_R)}. \quad (19)$$

By taking advantage of the results in (18) and (19), the outage probability can be approximated as:

$$\begin{aligned} P_{out}^R(M_R, L_R) &\simeq \hat{P}_{out}^R(M_R, L_R) = \\ &= \frac{1}{L_R!} \left(\frac{\beta L_R}{\bar{\rho}} \right)^{L_R 2^{-(1/M_R)}} \left(1 + \frac{\bar{\rho}}{\rho_t} \right)^{L_R 2^{-(1/M_R)} - 1}. \end{aligned} \quad (20)$$

Notice that outage of DR-OSTC (20) does not depend on the channel power unbalance ϵ . This key result is validated by the numerical analysis below and it can be exploited so as to ease the task of code design in distributed systems.

A. Numerical results

Here the outage probability approximation in (20) is corroborated by numerical simulations. As shown in figure 1, in the simulation setting the cooperating nodes are randomly deployed within a circular area (or cluster) of radius d_{cl} , the distance between the cluster center and the DN node is $d = 20d_{ref}$ with respect to a reference distance d_{ref} , say $d_{ref} = 1m$. Each average channel power $\Gamma_i = (d_i/d_{ref})^{-\alpha}$ takes into account a path loss term with exponent α , the distance d_i between the i -th cooperating node and the DN node is random and this makes Γ_i to be random as well. According to these choices, the probability density function $p_\Gamma(\Gamma)$ is evaluated numerically even if, in some cases, can be given analytically (e.g., for Alamouti with $L_D = L_R = 2$ [5]). For practical considerations, outage performance measurements

¹Notice that this is the most relevant case when distributed system is designed to be energy aware as it is the case when the network lifetime has to be preserved.

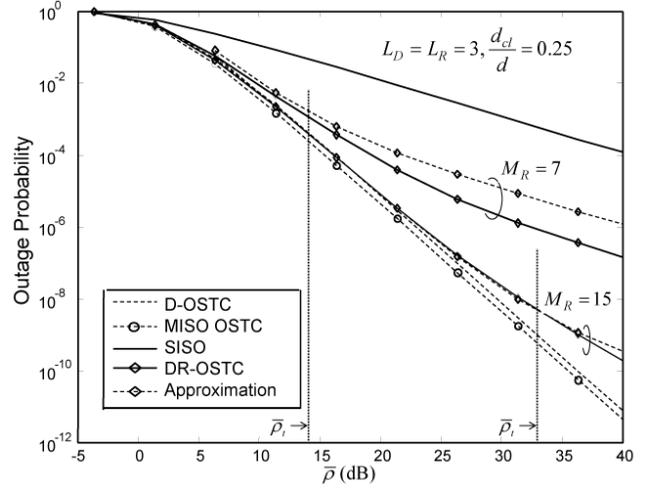


Fig. 3. Outage performances with respect to the average SNR $\bar{\rho}$ of Distributed Randomized OSTC (DR-OSTC) schemes with $M = 7$ and $M = 15$ cooperating terminals compared to D-OSTC and conventional OSTC (MISO OSTC lowerbound) with $L_D = L_R = 3$, $d_{cl}/d = 0.25$. Outage for DR-OSTC (20) is shown together with the SNR threshold $\bar{\rho}_t$ (18).

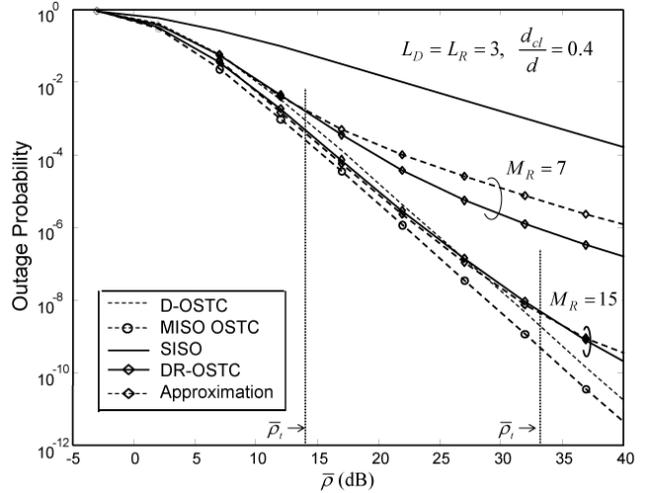


Fig. 4. Same setting as in fig. 3 with a wider spread of cooperating nodes: $d_{cl}/d = 0.4$.

are analyzed versus the ratio d_{cl}/d . Figure 2 shows that for each dispersion value d_{cl}/d and different path-loss exponents $\alpha = 2, 3, 4$, it follows the SNR degradation value ϵ according to (9). For realistic settings that limit the values of the ratio d_{cl}/d (say $0.05 \leq d_{cl}/d \leq 0.5$), numerical analysis shows that the SNR degradation ϵ is $\epsilon > 1 - d_{cl}/d$ (dashed line) whenever $\alpha \leq 3$.

Figure 3 and 4 show the outage probability for the DR-OSTC scheme with $\alpha = 3$, $\beta = 1$, $L_R = 3$, $d_{cl}/d = 0.25$ (figure 3) and $d_{cl}/d = 0.4$ (figure 4) versus the average SNR $\bar{\rho}$ for $M_R = 7$ and 15 cooperating nodes. Performances of MISO OSTC bound and its distributed version D-OSTC are shown as reference for $M_D = L_D = 3$ (recall that transmitted power for each node is scaled to highlight the diversity gain - see Sect. II). Outage probability approximation (dashed lines) in (20) is compared with the simulated performances (solid lines). For $\bar{\rho}$ values less than the threshold $\bar{\rho}_t$ (vertical lines

for both the $M_R = 7$ and 15 cases) the average outage probability decreases as if a finite SNR diversity of $L_R 2^{-1/M_R}$ could be achieved. Diversity converges to 1 as long as SNR $\bar{\rho} > \bar{\rho}_t$. Since the slope of the curve does not abruptly change from $L_R 2^{-1/M_R}$ to 1, when $\bar{\rho} > \bar{\rho}_t$ or $\bar{\rho} \simeq \bar{\rho}_t$ the proposed outage approximation (20) results in an upper bound with respect to the real performances. In contrast to the D-OSTC case, the SNR degradation ϵ (or equivalently d_{cl}/d ratio obtained from mapping in figure 2) has a minor impact on the outage performances of the DR-OSTC scheme. The coherent combination at the receiver of the signals belonging to the terminals that choose to serve as the virtual antenna array results in a substantial reduction of the fading power unbalancing with respect to the fully coordinated D-OSTC scheme.

V. CODE MATRIX DESIGN FOR DISTRIBUTED RANDOMIZED OSTC SCHEMES

Here it is developed the code design rules for the DR-OSTC protocol with any specified outage constraint. Although the number of cooperating nodes M_R or M_D is a random variable that depends on the broadcast phase duration, the transmit power allocation, the node and the fading distribution [16], here it is assumed that the number of decoding and cooperating nodes M_R or M_D is known by the source (or the DN) node (e.g., through a specific control channel). A discussion on more practical scenarios where the number of cooperating nodes is random and characterized by the probability density function (available at the transmitter) is in Sect. VI.

The design for DR-OSTC is comparative with respect to the D-OSTC protocol. In other words, the aim here is to find the minimum number of cooperating nodes for the DR-OSTC scheme \hat{M}_R , or the minimum required spatial dimension of the code \hat{L}_R , that should be employed to achieve the same outage performances of the D-OSTC strategy, in terms of a reliability requirement. At first, the D-OSTC design is carried out by selecting the complex orthogonal design \mathbf{C} for the D-OSTC scheme with dimensions $L_D \times p_D$ that meets a given link outage requirement \mathcal{P}_{out} (at any given rate R or threshold β). This is carried out by solving for L_D the inequality:

$$P_{out}^D(L_D) \leq \mathcal{P}_{out}. \quad (21)$$

According to (8), the spatial dimension L_D solution of (21) is a function of \mathcal{P}_{out} (and of the pair $\bar{\rho}, \epsilon$), therefore the outage requirement \mathcal{P}_{out} is mapped onto a required spatial dimension L_D of the D-OSTC space-time coding matrix. Next, for the L_D that follows from (21), a twofold design approach is proposed for the DR-OSTC that has the same outage as D-OSTC:

- For a given complex orthogonal design \mathbf{C} with dimension $L_R \geq L_D$ and such that the rate $R_{L_R} \geq R_{L_D}$, code design for the DR-OSTC scheme is dealt with in Sect V-A by defining M_R from the inequality:

$$P_{out}^R(M_R, L_R) \leq P_{out}^D(L_D). \quad (22)$$

Design rules will be held in the form:

$$M_R \geq \hat{M}_R(L_R, \epsilon, \bar{\rho}), \quad (23)$$

where $\hat{M}_R(L_R, \epsilon, \bar{\rho})$ is the minimum required number of cooperating nodes for any choice of L_R (at least $L_R \geq L_D$). Notice that outage probability requirement \mathcal{P}_{out} is embedded in L_R as $L_R \geq L_D$ (recall that L_D is solution to (21)). To guarantee a fair comparison with the D-OSTC scheme, the case of $L_R = L_D$ has been dealt with and analyzed separately. By substituting $L_R = L_D$ into (23), the minimum number of cooperating nodes becomes:

$$M_R \geq \hat{M}_R(L_D, \epsilon, \bar{\rho}). \quad (24)$$

For this case, being for a D-OSTC $M_D = L_D$, the quantity $\hat{M}_R - M_D > 0$ is the minimum number of cooperating nodes for DR-OSTC that should be added with respect to D-OSTC to meet the same outage requirement \mathcal{P}_{out} and simultaneously keep the same decoding complexity at the destination DN node (as $L_R = L_D$).

- For energy constrained networks where the number of decoding nodes is limited (e.g., by a maximum transmit power budget) to M_R and $M_R < \hat{M}_R(L_D, \epsilon, \bar{\rho})$, in Sect. V-B the code design for the DR-OSTC scheme relaxes the constraint $L_R = L_D$ by optimizing the spatial dimension \hat{L}_R (with $\hat{L}_R > L_D$) of the code matrix \mathbf{C} so that:

$$P_{out}^R(M_R, \hat{L}_R) = P_{out}^D(L_D), \text{ for } R_{L_R} \geq R_{L_D} \quad (25)$$

for any arbitrary number of cooperating terminals M_R . The problem is formulated by finding an orthogonal complex design \mathbf{C} whose code matrix spatial dimension \hat{L}_R satisfies (25) and thus meets the given outage probability requirement. Of course, the selected space-time code matrix \mathbf{C} should exhibit at least the same code rate R_{L_D} . A sufficient condition for the existence of the solution for any L_D is $R_{L_D} = 1/2$ (in this case a complex orthogonal design \mathbf{C} with $R_{\hat{L}_R} \geq 1/2$ exists for any value of \hat{L}_R [4]).

A. Minimum number of cooperating nodes given $L_R \geq L_D$

Let $P_{out}^D(L_D) \simeq \left(\frac{L_D \beta}{\bar{\rho} \epsilon}\right)^{L_D}$ be the outage probability for the OSTC when $\bar{\rho} \gg L_D \beta$ as in (8), by exploiting outage approximation (8) and (20), the inequality (22) reduces to:

$$\left(\frac{L_D \beta}{\bar{\rho} \epsilon}\right)^{L_D} \geq \left(\frac{L_R \beta}{\bar{\rho}}\right)^{L_R 2^{-(1/M_R)}} \left(1 + \frac{\bar{\rho}}{\rho_t}\right)^{L_R 2^{-(1/M_R)} - 1} \quad (26)$$

where in general $L_R \geq L_D$. Requirements for the minimum number of cooperating nodes satisfying (26), $\hat{M}_R(L_R, \epsilon, \bar{\rho})$, are derived in Appendix VIII-C (notice that in this case decoding complexity for DR-OSTC is higher than D-OSTC).

To ease the explanation, here we consider the particular case where the randomized space-time coding employs the same codeword matrix \mathbf{C} as for the D-OSTC, thus $L_R = L_D$. This allows a fair comparison between the two transmission strategies by preserving the same decoding complexity. By substituting $L_R = L_D$ into (26), after straightforward algebraic computations, M_R can be lower bounded by (see

Appendix VIII-C):

$$\begin{aligned} M_R &\geq \hat{M}_R(L_D, \epsilon, \bar{\rho}) = \\ &= \begin{cases} \hat{M}_R^{(1)}(L_D, \epsilon, \bar{\rho}), & \text{when } \epsilon \geq L_D^{-1/L_D} \\ 1 + (L_D - 1) \log_{L_D} \left(\frac{\epsilon \bar{\rho}}{\beta} \right) + \log_{L_D}(\epsilon), & \text{when } \epsilon < L_D^{-1/L_D} \end{cases} \quad (27) \end{aligned}$$

where

$$\hat{M}_R^{(1)} = \max \left\{ \left[\log_2 \left(\log_{\frac{\bar{\rho}(\epsilon)}{L_D \beta}} \left(\frac{\bar{\rho}}{L_D \beta} \right) \right) \right]^{-1}, 1 + (L_D - 1) \log_{L_D} \left(\frac{\bar{\rho}}{\beta} \right) \right\}. \quad (28)$$

B. Code matrix design given M_R

If the number of cooperating nodes is fixed (e.g., due to a transmit power constraint) to M_R , code design can be equivalently stated as finding the spatial dimension \hat{L}_R (that is $\hat{L}_R \geq L_D$) of the DR-OSTC code matrix \mathbf{C} ($\hat{L}_R \times p_R$) so that:

$$P_{out}^D(L_D) = P_{out}^R(M_R, \hat{L}_R) \simeq \hat{P}_{out}^R(M_R, \hat{L}_R), \quad (29)$$

with $R_{\hat{L}_R} \geq R_{L_D}$. Notice that, from the previous design setting, if $M_R < \hat{M}_R(L_D, \epsilon, \bar{\rho})$ then $\hat{L}_R > L_D$, in other words the space-time codeword dimension of the DR-OSTC should be larger with respect to D-OSTC to guarantee the same outage requirements. Solution might not be found when $R_{L_D} > 1/2$ as the maximum achievable code rate $R_{\hat{L}_R}$ decreases as \hat{L}_R increases. As a reference, in [23] it is shown that the maximum achievable code rate satisfies the rule $R_{\hat{L}_R} = (n+1)/2n$, where $n = \lfloor \hat{L}_R/2 \rfloor$.

Similarly as before for large SNR with $\bar{\rho} \gg \hat{L}_R \beta$ and by exploiting the outage approximation (20), the spatial dimension $\hat{L}_R > L_D$ should satisfy (26).

C. Code design examples

Here the outlined DR-OSTC design rules are corroborated by extensive numerical examples. In what follows it is employed a complex orthogonal design such that $R_{L_D} = 1/2$ (and $L_D > 1$). Figure 5 shows the outage probability for a D-OSTC scheme versus $\bar{\rho}$, with $d_{cl}/d = 0.2$ and 0.5 (see Sect. IV-A) and for $M_D = L_D = 2, 3$.

In figure 6 the minimum required number of cooperating nodes $\hat{M}_R(L_R, \epsilon, \bar{\rho})$ is investigated for the DR-OSTC scheme to achieve the same outage performances of the D-OSTC. The case $L_R = L_D$, in solid lines, accounts for the same decoding complexity at the receiver, the case $L_R = L_D + 1$ is in dashed lines. Design examples are discussed for varying d_{cl}/d (recall that SNR degradation ϵ is related to geometrical parameters d_{cl}/d from figure 2) ranging between 0.2 and 0.42, SNR $\bar{\rho} = 16\text{dB}$, 24dB and $L_D = 3$. The code design analysis reveals that the minimum required number of collaborating nodes can be reduced, in any case, by trading with decoding complexity at the receiver. The resulting settings are validated by simulating the outage performances from (7) and (17) (cross markers).

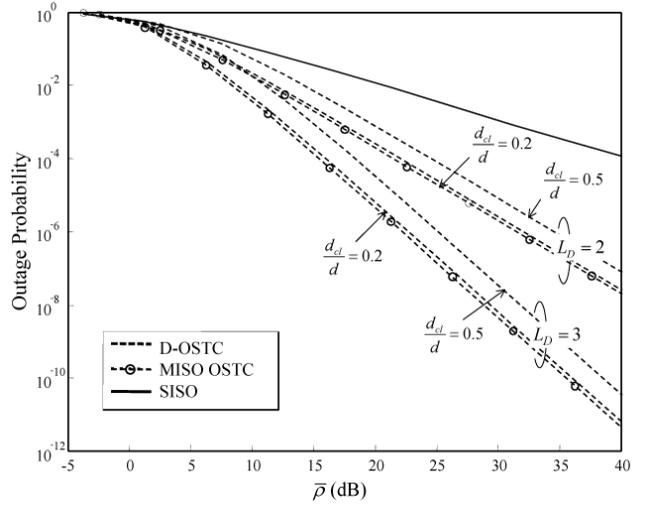


Fig. 5. Outage performances of D-OSTC with $L_D = M_D = \{2, 3\}$ versus the average SNR $\bar{\rho}$ for $d_{cl}/d = \{0.2, 0.5\}$ and $\alpha = 3$ (each d_{cl}/d value corresponds to a specific SNR degradation ϵ evaluated from figure 2). Conventional OSTC scheme (MISO OSTC lowerbound) and the single antenna case (SISO) are shown as reference.

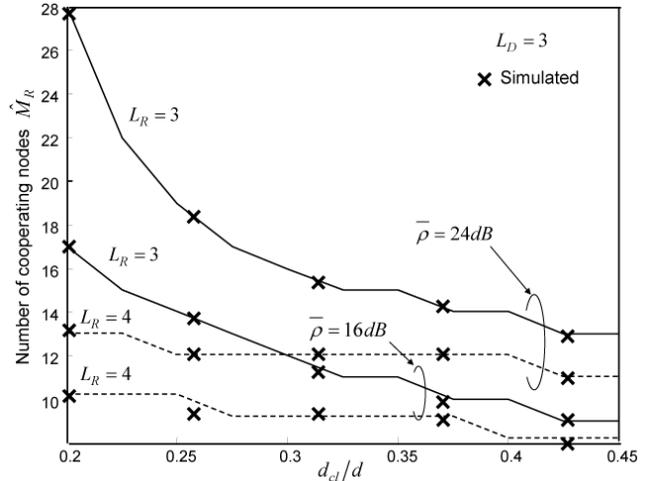


Fig. 6. Minimum required number of cooperating nodes, $\hat{M}_R(L_D, \epsilon, \bar{\rho})$, versus the ratio d_{cl}/d , for $L_D = 3$, $\bar{\rho} = 16\text{dB}$ and $\bar{\rho} = 24\text{dB}$: approximated results (solid lines for the case $L_R = L_D = 3$ and dashed lines for $L_R = L_D + 1 = 4$) obtained through the proposed design strategy (Sect. V) are compared with numerical simulations (cross markers).

For the case of $L_R = L_D = 3$ (fig. 6 - solid lines), the required number of relays $\hat{M}_R(L_D, \epsilon, \bar{\rho})$ is from (27): as the d_{cl}/d ratio decreases (or equivalently, ϵ approaches 1), performances of the D-OSTC in terms of outage probability converges to the lowerbound obtained with conventional OSTC (see fig. 5). For $\epsilon \rightarrow 1$ it is (from (28)):

$$\begin{aligned} \hat{M}_R(L_D, \epsilon, \bar{\rho}) &= \left[\log_2 \left(\log_{\frac{\bar{\rho}}{L_D \beta}} \left(\frac{\bar{\rho}}{L_D \beta} \right) \right) \right]^{-1} \simeq \\ &\simeq \frac{1}{1 - \epsilon} \log_2 \left(\frac{\bar{\rho}}{L_D \beta} \right), \quad (30) \end{aligned}$$

the number of required cooperating nodes for the DR-OSTC increases with $1/(1-\epsilon)$. On the contrary, when the propagation environment is such that (from (27)) $\epsilon < L_D^{-1/L_D} \simeq 0.69$

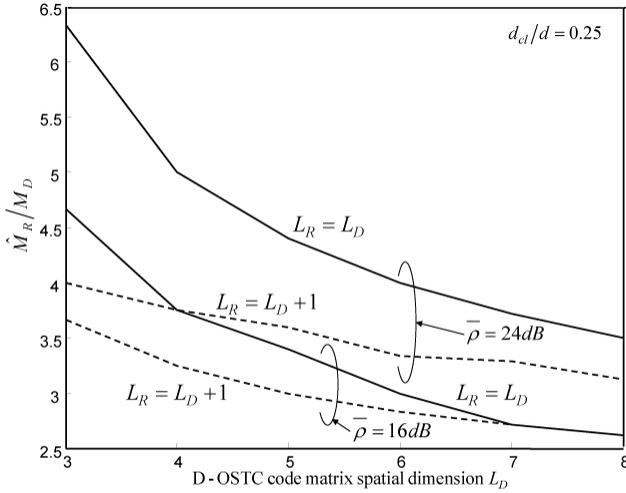


Fig. 7. Increase factor for the number of required cooperating nodes (defined as the ratio between the required number of nodes - $\hat{M}_R(L_D, \epsilon, \bar{\rho})$ (solid lines) or $\hat{M}_R(L_D + 1, \epsilon, \bar{\rho})$ (dashed lines) - for the DR-OSTC and L_D) versus the D-OSTC code matrix spatial dimension L_D . Both $\bar{\rho} = 16\text{dB}$ and 24dB cases are considered ($d_{cl}/d = 0.25$).

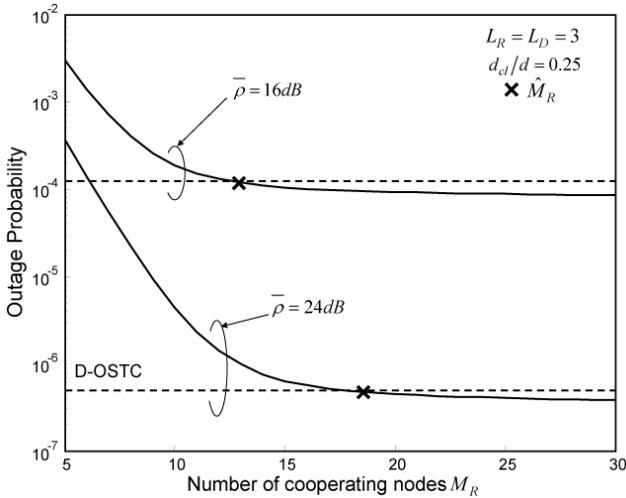


Fig. 8. Outage probability (20) for the DR-OSTC scheme with respect to the number of cooperating nodes M_R when $\bar{\rho} = 16\text{dB}$ and 24dB ($d_{cl}/d = 0.25$). Dashed horizontal lines refer to the outage performances for the D-OSTC case with $L_D = 3$. We consider the case $L_R = L_D = 3$, the required number of cooperating nodes, \hat{M}_R is indicated by cross markers for all cases (same setting as in figure 6).

(or for large cluster size as $d_{cl}/d \simeq 0.36$, and $L_D = 3$), requirements on M_R become less stringent.

For the case of $L_R = L_D + 1 = 4$ (fig. 6 - dashed lines), the minimum required number of cooperating nodes, $\hat{M}_R(L_R, \epsilon, \bar{\rho})$, satisfies (26). The code spatial dimension L_R is greater than L_D to benefit from higher diversity at the price of an higher decoding complexity of DR-OSTC with respect to D-OSTC. The minimum required number of cooperating nodes decreases for any value of ϵ when compared to the case $L_R = L_D = 3$ (solid lines). As a simple argument to justify this behavior, notice that, for any $M_R \geq L_D$, when the code spatial dimension of the DR-OSTC scheme L_R increases, the probability that the number of code matrix \mathbf{C} rows selected

by the M_R cooperating nodes (see (39)) is larger than L_D is $\Pr(i \geq L_D) = 1 - \binom{L_R}{L_R - L_D + 1} \left(\frac{L_D - 1}{L_R}\right)^{M_R} \sim 1$. This latter condition guarantees that at least the same diversity degree L_D of the D-OSTC protocol is achieved by the randomized scheme. A complexity trade-off between the minimum required number of cooperating nodes M_R and the required decoding complexity at the receiving node has to be accounted for. Comparing solid and dashed lines in figure 6, it turns out that, by letting $L_R = L_D + 1$ (thus by increasing the spatial dimension of the space-time code by one), the minimum number of cooperating nodes $\hat{M}_R(L_D + 1, \epsilon, \bar{\rho})$ is reduced approximately by half value with respect to $\hat{M}_R(L_D, \epsilon, \bar{\rho})$.

For the sake of completeness, figure 7 shows how the required number of cooperating nodes of DR-OSTC is affected for an increasing L_D . The ratio between the number of required cooperating nodes for DR-OSTC, \hat{M}_R (both with $L_R = L_D$ and $L_R = L_D + 1$) and the required nodes for the D-OSTC scheme, $M_D = L_D$, is visualized for various required diversity degrees L_D .

VI. COOPERATIVE PROTOCOLS DESIGN WITH DR-OSTC SCHEME

In this section the outage constrained code design developed so far is tailored for a more realistic environment where the number of decoding relays is random due to propagation environment and random node distribution. By analyzing the two cooperative protocols outlined in Sect. I-A, we develop simple design rules that can be effectively implemented to guarantee the required system performances.

In realistic propagation environments, due to mobility of terminals and decoding errors at the relays, the number of active nodes that successfully decode and collaborate during the cooperative transmission is not known when the space-time code matrix is designed. As an example, when considering the fully coordinated D-OSTC protocol, if one of the L_D selected nodes of the cooperative set fails in decoding or becomes inactive (due to high node mobility), performances (in terms of outage or BER probability) would loose the full degree of diversity and this cannot be tolerated in some cases. As a consequence, to guarantee a certain level of robustness, the protocol requires an acknowledgment phase from the collaborating nodes before the mapping set-up. This overhead could severely affect the overall packet delivery delay and thus the system throughput. On the other hand, a DR-OSTC protocol neither requires the knowledge of which node is going to collaborate, nor an acknowledgement phase that would increase the MAC layer complexity. The robustness of the DR-OSTC scheme with respect to the number of cooperating nodes is shown in figure 8. The outage probability obtained from approximation (20) for the DR-OSTC scheme is analyzed with respect to the number of cooperating nodes M_R (for $d_{cl}/d = 0.25$) when $\bar{\rho} = 16\text{dB}$ and 24dB ; the analysis here is limited to the case $L_R = L_D$, (24) (for $L_D = 3$). Dashed horizontal lines refer to the outage performances for the D-OSTC case with $M_D = L_D = 3$. The minimum number of cooperating nodes $\hat{M}_R(L_D, \epsilon, \bar{\rho})$ can be obtained from figure 6 (for $d_{cl}/d = 0.25$) and it is indicated by cross markers. We notice that, although an higher number of

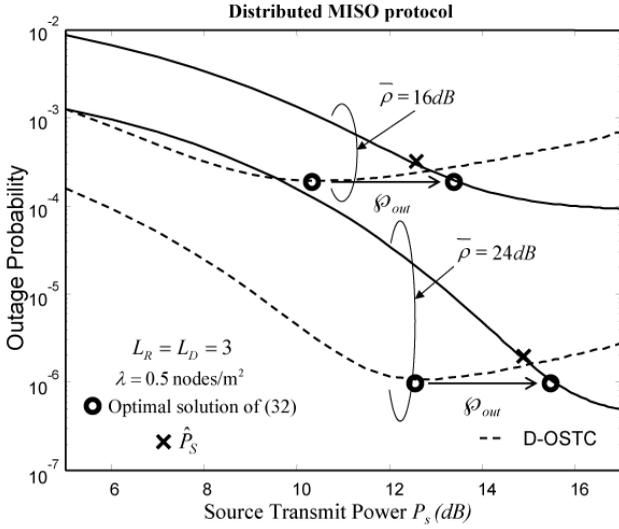


Fig. 9. Outage probability at the destination BS for the DR-OSTC scheme ($L_R = L_D$) with respect to the source transmit power P_S in case of a distributed MISO protocol. Neighbor nodes within the transmission cluster are distributed according to a Poisson random point process. Dashed lines refer to the outage performances for the D-OSTC case with $L_D = 3$ while outage probability requirement \mathcal{P}_{out} is indicated by circular markers. Lower bound \hat{P}_S to satisfy the same outage \mathcal{P}_{out} for DR-OSTC is indicated by cross markers, while circular markers refer to the optimal solution to (32).

cooperating nodes is required with respect to the D-OSTC scheme, in practice, it can be substantially reduced with respect to $\hat{M}_R(L_D, \epsilon, \bar{\rho})$ at the price of a negligible outage probability increasing. This shows that DR-OSTC is not too sensitive to M_R if $M_R \sim \hat{M}_R$.

A. Case study for Poisson random networks in short range wireless environments

For completeness, starting from the previous results, we now assess the performance of the DR-OSTC scheme when applied to both the distributed MISO and the incremental relaying protocols outlined in Sect I-A. To model the random number of decoding (or cooperating) nodes it is adopted a “disk model” for the links between the transmitter and the relays. This model is known to be suited for short range wireless applications [14] where the fading condition is favorable due to a dominant line of sight component. Notice that for the link between source and relays, the stochastic nature of the fading channel (and thus the fact that the SNR is a random variable) is neglected even if it would be straightforward to consider more complex schemes.

According to the disk model, a successful transmission to a relay at distance d_r occurs as long as the received SNR exceeds the threshold β : $P_S d_r^{-\alpha} > \beta$, where P_S is the broadcast power level (normalized with respect to the unit power AWGN noise), in general $P_S \neq P_r$. The node range r_g for successful transmission is $r_g = (P_S/\beta)^{1/\alpha}$ and, according to the geometrical model illustrated in Sect. IV-A, the radius of the cluster of cooperating nodes is $d_{cl} = r_g$.

Let the terminals be distributed in the plane according to a Poisson random point process with node density λ , the probability of M successfully decoding (and thus cooperating)

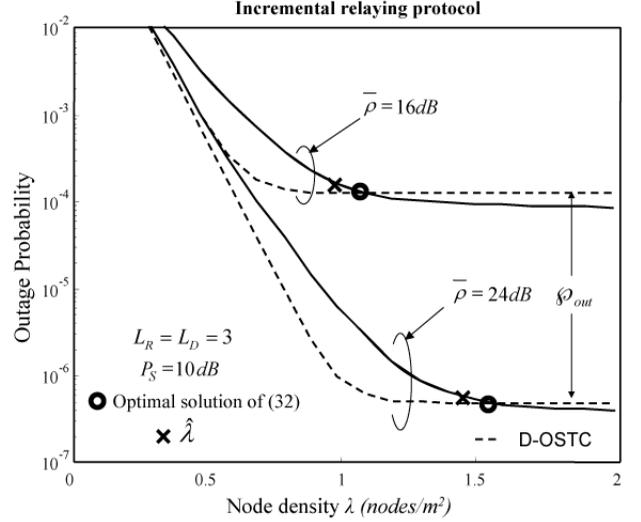


Fig. 10. Outage probability at the destination BS for the DR-OSTC scheme ($L_R = L_D$) with respect to the local node density λ in case of an incremental relaying protocol. Neighbor nodes within the transmission cluster are distributed according to a Poisson random point process. Dashed lines refer to the outage performances for the D-OSTC case with $L_D = 3$ while outage probability requirement \mathcal{P}_{out} is indicated by circular markers. Lower bound $\hat{\lambda}$ to satisfy the same outage \mathcal{P}_{out} for DR-OSTC is indicated by cross markers, while circular markers refer to the optimal solution to (32).

nodes within the area $\mathcal{A} = \pi r_g^2$ is:

$$P_{\mathcal{A}}(M_R) = \exp(-\lambda \pi r_g^2) \frac{(\lambda \pi r_g^2)^{M_R}}{M_R!} \quad (31)$$

with average number of decoding nodes $E_{P_{\mathcal{A}}}[M_R] = \lambda \pi r_g^2 = \lambda \pi (P_S/\beta)^{2/\alpha}$. The distributed MISO protocol requires a broadcast phase where the transmit power of the source node P_S is designed to meet the (average) outage probability requirement \mathcal{P}_{out} at the DN when employing the cooperative transmission protocol:

$$\sum_{M_R=0}^{\infty} P_{out}^R(M_R + 1, L_R) P_{\mathcal{A}}(M_R) = \mathcal{P}_{out}, \quad (32)$$

where $P_{out}^R(M_R, L_R) \simeq \hat{P}_{out}^R(M_R, L_R)$ is the outage for a space-time coding matrix (20) with $L_R = L_D$ (recall that L_D is fixed and solution to (21)) and the weighted summation comes from the total probability law. By taking advantage of the results in Sect. V, and selecting the power level P_S such that the average number of decoding relays, $\lambda \pi (P_S/\beta)^{2/\alpha}$, equals the minimum $\hat{M}_R(L_D, \epsilon, \bar{\rho}) - 1$ (we assume the source node is collaborating, even if other strategies may be employed as well), it yields the lower bound \hat{P}_S on power as:

$$P_S \geq \hat{P}_S = \beta \left(\frac{\hat{M}_R(L_D, \epsilon, \bar{\rho}) - 1}{\pi \lambda} \right)^{\alpha/2}, \quad (33)$$

we refer to Appendix VIII-D for a detailed proof. Notice that the minimum power level \hat{P}_S can be derived at the source node with minimal signalling exchange with the DN based on the knowledge of the local neighbor node distribution (e.g., the local density λ for a Poisson random network), the required diversity order L_D , the performance degradation

$\epsilon > 1 - d_{cl}/d$, see Sect. IV-A, (such that condition (21) holds) and the average SNR $\bar{\rho}$.

When considering the incremental relaying protocol, the source power is fixed so as to guarantee a given link reliability towards the DN. In this case the design should be focused on the minimum node density λ to guarantee an (average) outage probability at the DN \mathcal{P}_{out} after the cooperative retransmission (perfect feedback channel from DN to the node cluster is assumed, one single retransmission phase is allowed by the system). Similarly as before, the node density has to be designed so as to satisfy (32), moreover it is viable the evaluation of a lower bound for λ :

$$\lambda \geq \hat{\lambda} = \frac{\hat{M}_R(L_D, \epsilon, \bar{\rho})}{\pi} \left(\frac{\beta}{P_S} \right)^{2/\alpha}, \quad (34)$$

details can be found in Appendix VIII-D. The evaluation of lowerbound $\hat{\lambda}$ allows the source node to decide whether or not the node density is enough for a cooperative retransmission to guarantee the required outage reliability level \mathcal{P}_{out} .

In figure 9 and 10 we assess the tightness of the proposed lower bounds for both the cooperative protocols considered here. We show the outage probability at the destination DN node for a Poisson random network obtained from the summation in (32) with $P_{out}^R(M_R, L_R) \simeq \hat{P}_{out}^R(M_R, L_R)$. On figure 9 the required transmit power P_S at the source node during the broadcast phase of the distributed MISO protocol is investigated. In figure 10 the analysis focuses on the required node density according to the incremental relaying protocol. Results are shown for $L_R = L_D = 3$ and for various $\bar{\rho}$ values (the cluster radius d_{cl} is defined according to the source transmit power P_S), cross markers refer to the minimum source power \hat{P}_S (on figure 9) and node density $\hat{\lambda}$ (on figure 10). As before, dashed lines refer to the outage performances for the D-OSTC case where the maximum number of cooperating nodes is constrained to $M_D = L_D$. Notice that, regardless of the cooperative protocol, a low source power level P_S or node density λ would result in a lack of cooperating nodes and thus performance loss. Moreover, being diversity provided by the ST code limited to L_D from (21), a large source power P_S would cause performance degradation due to the increased cluster size (and SNR degradation ϵ). Although the DR-OSTC scheme requires an higher demand of network resources with respect to D-OSTC (in terms of an increased source power or node density), it does not need any acknowledgement phase among the collaborating relay nodes before starting the cooperative transmission and this results in a significant MAC layer complexity reduction (together with a reduced packet delivery delay). In both cases the bounds $P_S = \hat{P}_S$ and $\lambda = \hat{\lambda}$ are tight in estimating the optimal solutions (circular markers) for the outage probability requirement \mathcal{P}_{out} . The latter result can be motivated by recalling that when $L_R = L_D$ the number of cooperating nodes can be substantially reduced with respect to $\hat{M}_R(L_D, \epsilon, \bar{\rho})$ with still a negligible performance degradation (see figure 8). Since small outage performance loss with respect to the optimal solution (32) can be experienced, the results in (33) and (34) are useful in designing practical cooperating protocols.

VII. CONCLUDING REMARKS

This paper considered two cooperative protocols where transmission of symbols (corrupted symbols in case of incremental relaying) is relayed by a number of cooperating nodes employing a distributed orthogonal space time coding scheme. Outage performances of the system have been analyzed assuming a Distributed Randomized Orthogonal Space-Time Coding scheme (DR-OSTC) to be employed by the relaying terminals during the cooperative transmission session. Within this class, the focus is on the randomized antenna selection scheme that requires each cooperating node to choose *randomly* and *independently* to serve as one of the space-time virtual antennas. The DR-OSTC has been compared with the distributed space-time coding schemes (D-OSTC) and the conventional multiple antenna based OSTC. Moreover, simple and accurate design rules have been provided for the required minimum number of cooperating nodes \hat{M}_R (Sect. V) and for the space-time code matrix spatial dimension \hat{L}_R of DR-OSTC so as to meet a specific outage probability requirement at the destination node. A trade off between the number of sensors M_R and spatial dimension L_R should be exploited as the requirements on M_R might be less stringent when increasing the spatial dimension ($L_R > L_D$) of the complex orthogonal design (resulting, however, in a decoding complexity increase at the DN node).

Due to its inherent distributed structure, the DR-OSTC has proved to significantly reduce the required amount of control overhead (if compared to the D-OSTC scheme), as it does not need any pre-defined “terminal-to-code matrix row mapping”, nor an acknowledgement phase among the collaborating nodes. However, this property can be accomplished at the price of an increased number of cooperating nodes when compared with D-OSTC that requires careful design of the source transmit power (when considering the distributed MISO protocol) or the minimum sensor density within a given deployment area (when considering the incremental relaying). By noticing that, when $L_R = L_D$ the outage performances are still maintained even if some nodes cannot cooperate due to decoding failures, protocol design rules are also given for practical environments.

Since the DR-OSTC protocol substantially reduces the MAC layer complexity as it allows for a full distributed implementation of the space-time coding among the collaborative terminals, this scheme offers a valuable solution for future cooperative wireless communication systems.

VIII. APPENDIX

A. Diversity and SNR loss for the D-OSTC

For a given vector $\mathbf{\Gamma}_{L_D}$, assuming $\bar{\rho} \gg L_D\beta$, the outage probability can be approximated as [22]:

$$\begin{aligned} P_{out}^D(L_D | \mathbf{\Gamma}_{L_D}) &= \\ &= \sum_{i=1}^{L_D} A_i(\mathbf{\Gamma}_{L_D}) \left(1 - \exp\left(-\frac{\beta L_D}{\Gamma_i P_r}\right) \right) \simeq \frac{\left(\frac{\beta L_D}{P_r}\right)^{L_D}}{L_D! \prod_{i=1}^{L_D} \Gamma_i}. \end{aligned}$$

By averaging with respect to the L_D i.i.d. fading powers and recalling that $\bar{\rho} = P_r E_\Gamma[\Gamma]$, the outage probability becomes:

$$P_{out}^D(L_D) = \frac{1}{L_D!} \left(\frac{\beta L_D E_\Gamma[\Gamma]}{\bar{\rho}} \right)^{L_D} \prod_{i=1}^{L_D} \int \frac{p_\Gamma(\Gamma_i)}{\Gamma_i} d\Gamma_i = P_{out}(L_D) \left(E_\Gamma[\Gamma] E_\Gamma \left[\frac{1}{\Gamma} \right] \right)^{L_D}, \quad (35)$$

where $P_{out}(L_D)$ is the outage probability for a conventional OSTC scheme that for $\bar{\rho} \gg L_D \beta$ reduces as [22]:

$$P_{out}(L_D) \simeq \frac{1}{L_D!} \left(\frac{L_D \beta}{\bar{\rho}} \right)^{L_D}. \quad (36)$$

In (35) $(E_\Gamma[\Gamma] \cdot E_\Gamma[\frac{1}{\Gamma}])^{L_D}$ is the performance loss due to unequal average fading powers. We may now rewrite (35) as in (8) by highlighting the SNR loss due to the distributed space-time scheme.

Notice that, for any given (deterministic) vector $\mathbf{\Gamma}_{L_D}$, being, in this case, $\bar{\rho} = P_r \left(\sum_{i=1}^{L_D} \Gamma_i \right) / L_D$ and $\bar{\rho} \gg L_D \beta$

$$P_{out}^D(L_D | \mathbf{\Gamma}_{L_D}) \simeq \frac{1}{L_D!} \left(\frac{\beta L_D}{\bar{\rho}} \right)^{L_D} \frac{\left(\sum_{i=1}^{L_D} \Gamma_i \right)^{L_D}}{L_D^{L_D} \prod_{i=1}^{L_D} \Gamma_i}, \quad (37)$$

thus, similarly as in (8) we may write:

$$P_{out}^D(L_D | \mathbf{\Gamma}_{L_D}) \simeq \frac{1}{L_D!} \left(\frac{\beta L_D}{\bar{\rho} \epsilon(\mathbf{\Gamma}_{L_D})} \right)^{L_D} \quad (38)$$

where $\epsilon(\mathbf{\Gamma}_{L_D}) = \left[\left(L_D \sqrt[L_D]{\prod_{i=1}^{L_D} \Gamma_i} \right) / \sum_{i=1}^{L_D} \Gamma_i \right]$ models the SNR loss for any arbitrary selection of $\mathbf{\Gamma}_{L_D}$ vector.

B. DR-OSTC

From the multinomial analysis proposed in Sect. IV we define:

$$\begin{aligned} \Pr(i) &= \Pr(m=i) = \\ &= \binom{L_R}{L_R-i} \left(\frac{i}{L_R} \right)^{M_R} - \binom{L_R}{L_R-i+1} \left(\frac{i-1}{L_R} \right)^{M_R} \end{aligned} \quad (39)$$

as the probability that i (with $i \leq L_R$) rows of code matrix \mathbf{C} are selected by the M_R cooperating nodes. Assuming $\bar{\rho} = P_r E_\Gamma[\Gamma] \gg L_R \beta$ and $M_R > L_R$ such that $\tilde{\Gamma}_i = \sum_{m=1}^{M_R} g_{i,m} \Gamma_m \simeq k_i E_\Gamma[\Gamma]$, $i = 1, \dots, L_R$, (almost sure convergence is guaranteed as long as $M_R \rightarrow \infty$) we derive a lower bound on the outage probability:

$$\begin{aligned} P_{out}^R(M_R, L_R) &> \\ &> \int p_\Gamma(\mathbf{\Gamma}_{M_R}) \sum_{i=1}^{L_R} \frac{\Pr(i) M_R^i \beta^i}{i!} \left(P_r \sum_{m=1}^{M_R} g_{i,m} \Gamma_m \right)^{-i} d\mathbf{\Gamma}_{M_R} \\ &> \sum_{i=1}^{L_R} \frac{\Pr(i)}{i!} \left(\frac{M_R \beta}{\bar{\rho}} \right)^i \left[E_{\mathbf{k}_i} \left(\prod_{p=1}^i k_p \right) \right]^{-1} > \\ &> \sum_{i=1}^{L_R} \Pr(i) \left(\frac{i \beta}{\bar{\rho}} \right)^i \frac{1}{i!} \end{aligned} \quad (40)$$

where $E_{\mathbf{k}_i} \left(\prod_{p=1}^i k_p \right)$ is the average value of the product $\prod_{p=1}^i k_p$ over all the possible combinations $\mathbf{k}_i = [k_1, \dots, k_i]^T$ such that $\sum_{p=1}^i k_p = M_R$. For each i , $E_{\mathbf{k}_i} \left(\prod_{p=1}^i k_p \right)$ can be upperbounded by the combination that gives the maximum diversity of i , thus $E_{\mathbf{k}_i} \left(\prod_{p=1}^i k_p \right) < \left(\frac{M_R}{i} \right)^i$.

The diversity order for any finite value of M_R is 1 as

$$\begin{aligned} \lim_{\bar{\rho} \rightarrow \infty} \frac{-\log(P_{out}^R(M_R, L_R))}{\log(\bar{\rho})} < \\ < \frac{-\log \left(\sum_{i=1}^{L_R} \Pr(i) \left(\frac{i \beta}{\bar{\rho}} \right)^i \frac{1}{i!} \right)}{\log(\bar{\rho})} = 1. \end{aligned} \quad (41)$$

However, when M_R is large such that

$$\Pr(1) = \left(\frac{1}{L_R} \right)^{M_R-1} \leq \left(\frac{\beta}{\bar{\rho}} \right)^{L_R-1} \quad (42)$$

then, using the result in (40) and the Jensen inequality, the diversity performance at finite $\bar{\rho}$ [21] (but still $\bar{\rho} \gg L_R \beta$) can be upperbounded as:

$$\frac{-\log(P_{out}^R(M_R, L_R))}{\log(\bar{\rho})} < \sum_{i=1}^{L_R} i \Pr(i) < L_R 2^{-\frac{1}{M_R}}. \quad (43)$$

Inequality (42) is satisfied as long as:

$$\bar{\rho} < \bar{\rho}_t = \beta L_R^{\frac{M_R-1}{L_R-1}} \text{ or } M_R > 1 + (L_R - 1) \log_{L_R} \left(\frac{\bar{\rho}}{\beta} \right) \quad (44)$$

Exploiting results (41), (43) and (44), outage curve versus $\bar{\rho}$ ($\bar{\rho} \gg L_R \beta$) can be thus approximated as:

$$\begin{aligned} P_{out}^R(M_R, L_R) &\simeq \hat{P}_{out}^R(M_R, L_R) = \\ &= \frac{1}{L_R!} \left(\frac{L_R \beta}{\bar{\rho}} \right)^{\frac{L_R}{2^{1/M_R}}} \left(1 + \frac{\bar{\rho}}{\bar{\rho}_t} \right)^{\left(\frac{L_R}{2^{1/M_R}} \right)^{-1}}. \end{aligned} \quad (45)$$

C. Required minimum number of cooperating nodes \hat{M}_R when $L_R \geq L_D$

When $\bar{\rho} < \bar{\rho}_t = \beta L_R^{\frac{M_R-1}{L_R-1}}$ equation (26) becomes

$$\left(\frac{L_D \beta}{\epsilon \bar{\rho}} \right)^{L_D} \geq \left(\frac{L_R \beta}{\bar{\rho}} \right)^{\frac{L_R}{2^{1/M_R}}}, \quad (46)$$

after straightforward algebraic computations, M_R can be designed as:

$$\begin{aligned} M_R &\geq \hat{M}_R^{\bar{\rho} < \bar{\rho}_t}(L_R, \epsilon, \bar{\rho}) = \\ &= \max \left\{ \left(\log_2 \left[\left(\frac{L_R}{L_D} \right) \log_{\frac{\bar{\rho} \epsilon}{L_D \beta}} \left(\frac{\bar{\rho}}{L_R \beta} \right) \right] \right)^{-1}, \right. \\ &\quad \left. 1 + (L_R - 1) \log_{L_R} (\bar{\rho} / \beta) \right\}. \end{aligned} \quad (47)$$

When $\bar{\rho} > \bar{\rho}_t = \beta L_R^{\frac{M_R-1}{L_R-1}}$ inequality (26) reduces to:

$$\left(\frac{L_D \beta}{\epsilon \bar{\rho}} \right)^{L_D} \geq \left[\frac{\beta}{\bar{\rho}_t} \right]^{\left(\frac{L_R}{2^{1/M_R}} \right)^{-1}} \left(\beta L_R^{\left(\frac{L_R}{2^{1/M_R}} \right)} / \bar{\rho} \right), \quad (48)$$

thus M_R can be numerically derived (assuming $M\sqrt{2} \simeq 1$) by solving:

$$\begin{cases} M_R > 1 + (L_D - 1) \log_{L_R} \left(\frac{\epsilon \bar{\rho}}{\beta} \right) + \log_{L_R} \left(\epsilon L_R^{L_R} / L_D^{L_D} \right) \\ M_R < 1 + (L_R - 1) \log_{L_R} \left(\frac{\bar{\rho}}{\beta} \right) \end{cases} \quad (49)$$

By combining conditions (47) and (48), the minimum number of cooperating nodes reads:

$$\begin{aligned} M_R &\geq \hat{M}_R(L_R, \epsilon, \bar{\rho}) = \\ &= \begin{cases} \hat{M}_R^{\bar{\rho} < \bar{\rho}^t}(L_R, \epsilon, \bar{\rho}), & \text{when } \epsilon \geq (L_R)^{-(1+K)/L_D} \\ 1 + (L_D - 1) \log_{L_R} \left(\frac{\epsilon \bar{\rho}}{\beta} \right) + \log_{L_R} \left(\epsilon L_R^{L_R} / L_D^{L_D} \right), & \text{when } \epsilon < (L_R)^{-(1+K)/L_D} \end{cases} \end{aligned}$$

where $K = \log_{L_R} \left(L_R^{L_R} / L_D^{L_D} \right) - (L_R - L_D) \log_{L_R} (\bar{\rho} / \beta)$.

D. Source power and node density design

Although equations (32) can be easily solved for both P_S and λ , by numerical analysis (even for a generic probability density function $P_A(M_R)$), here a simple (and practical) lower bound to the solution is derived. By using the Jensen inequality and by noticing that the outage probability curve is convex with respect to the number of cooperating nodes (see figure 8), equation (32) can be simplified to:

$$\mathcal{P}_{out} > \hat{P}_{out}^R(E_{P_A}[M_R], L_R) \quad (50)$$

By following the same steps as in Section V, the average value $E_{P_A}[M_R]$ has to be constrained so that

$$E_{P_A}[M_R] > \Psi, \quad (51)$$

where $\Psi = \hat{M}_R(L_D, \epsilon, \bar{\rho}) - 1$ for the distributed MISO protocol and $\Psi = \hat{M}_R(L_D, \epsilon, \bar{\rho})$ for the incremental relaying protocol. Assuming a Poisson random network where the disk model applies, the design rule in (51) can be simplified into:

$$\lambda \pi \sqrt{(P_S / \beta)^2} > \Psi, \quad (52)$$

by solving with respect to the source power P_S or the local node density λ we obtain (33) and (34), respectively.

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